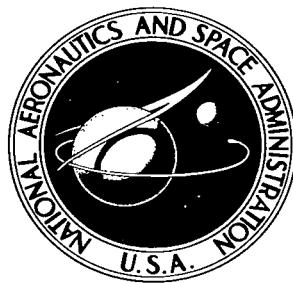


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**PLANETARY PERTURBATIONS
IN CHEBYSHEV SERIES**

by Lloyd Carpenter

*Goddard Space Flight Center
Greenbelt, Md.*

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C.



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ABSTRACT

The practical value of using Chebyshev series in solving the equations of planetary motion is demonstrated. Solutions are obtained for the mutual attractions of the five outer planets. Residuals from comparisons with numerical integrations are less than $0.^{\circ}001$ in all cases for the two-century interval from 1800 through 2000. A listing is given of the FORTRAN IV program used for the computations. The first order general perturbations of the outer planets due to the inner planets are given for completeness.

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PLANETARY PERTURBATIONS IN CHEBYSHEV SERIES

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Lloyd Carpenter
Goddard Space Flight Center

INTRODUCTION

The exploration of space has given impetus to studies of the motions of planets and the advent of modern computers has contributed greatly to the success of these works. The greatest success has been in the realm of numerical integration, and the outstanding example is the application to the motions of the five outer planets over a period of more than 120,000 years by Cohen and Hubbard (Reference 1). Their work revealed the librational motions of Pluto and Neptune, a truly remarkable result.

Considerable effort has also been directed to the computation of general planetary perturbations (series expansions). The prime example is Clemence's Theory of Mars (Reference 2 and 3), the first planetary theory which is formally more accurate than the observations. The works of Iszak (References 4, 5, and 6) on literal developments, of Danby (References 7, 8, 9, 10, and 11) using matrix methods, and of Musen (References 12, 13, 14, and 15) using operator calculus and Pfaff's method are noteworthy. The author (References 16, 17, 18, and 19) has also been interested in the computation of general planetary perturbations.

Accurate solutions of the equations of planetary motion are important for producing ephemerides, for identification in the case of minor planets, and for improvement of the values of the astronomical constants through analyses of observations. Clemence (Reference 20) has shown the value of analysis of minor planet observations for the latter purpose when a formally accurate solution of the equations is available. The purpose of the present work is to present a method which may be of particular value in the case of the minor planets and which is somewhat different from the usual special or general perturbations. To establish the accuracy of the results and to provide a basis for computing perturbations of minor planets, the method has first been applied to the five outer planets. The results are, in fact, the most accurate planetary theories ever produced, as is demonstrated by comparisons with numerical integration yielding residuals less than $0''.001$ in all cases. In this context, a "planetary theory" is defined, in the words of Danby (Reference 11), to be "a mathematical expression that yields an ephemeris of a planet over a finite time." The perturbations are computed using series expansions in the well-known trigonometric polynomials of Chebyshev (Reference 21). In the examples, the expressions are valid from 1800 through 2000. The method can be applied, in principle, with any specified time interval, but the rate of convergence of the series depends on the length of the time interval.

The method of Chebyshev series does not suffer from all the disadvantages common to methods of general planetary perturbations. The intersection of the orbit of the disturbed planet with the orbit of a disturbing planet presents no difficulty unless there is an actual close approach of the two planets. Commensurability of the motions does not require any special consideration. These facts are illustrated in the motions of Neptune and Pluto. The computation of the perturbations in Chebyshev series is much simpler than for the classical methods of general perturbations. The program described in this article actually computes all terms of all orders in the mutual attraction of a system of planets giving accuracies never before obtained in a series solution. Finally, the concise, homogeneous form of the solution is most convenient. With the new generation of computers becoming available, the method can be easily applied to many interesting problems.

CHEBYSHEV POLYNOMIALS

An excellent detailed discussion of Chebyshev polynomials is given by Lanczos (Reference 22). Additional properties and applications are given in References 23, 24, and 25. Only the required basic relationships are given here.

The Chebyshev polynomial $T_n(x)$ may be defined by

$$T_n(x) = \cos[n \cos^{-1} x]$$

for

$$-1 \leq x \leq +1 .$$

The formulas for manipulating Chebyshev polynomials can be derived from the well-known trigonometric formulas. $T_n(x)$ is a polynomial of degree n in x and can be generated by the recurrence relation

$$T_0(x) = 1 ,$$

$$T_1(x) = x ,$$

$$T_n(x) = 2x T_{n-1}(x) - T_{n-2}(x) .$$

The product of two Chebyshev polynomials is given by

$$2T_r(x) T_q(x) = T_{r+q}(x) + T_{|r-q|}(x) ,$$

and the integral is given by

$$\int T_0(x) dx = T_1(x) ,$$

$$\int T_1(x) dx = \frac{1}{4} T_2(x) ,$$

$$2 \int T_r(x) dx = \frac{1}{r+1} T_{r+1}(x) - \frac{1}{r-1} T_{r-1}(x) , \quad r > 1 .$$

Any function $f(x)$ which is continuous and of bounded variation in the interval $-1 \leq x \leq +1$ can be represented by a uniformly convergent series of Chebyshev polynomials in that interval,

$$f(x) = \sum_{r=0}^{\infty}' a_r T_r(x)$$

for

$$-1 \leq x \leq +1$$

where the prime on the summation sign is used to indicate that the first term is to be factored by one-half,

$$f(x) = \frac{1}{2} a_0 + a_1 T_1(x) + a_2 T_2(x) + \cdots .$$

Approximate values, \bar{a}_r , of the coefficients can be determined from special values of the function using the formula

$$\bar{a}_r = \frac{2}{N} \sum_{s=0}^{N-1} f\left(\cos \frac{s\pi}{N}\right) \cos \frac{rs\pi}{N} ; \quad r = 0, 1, 2, \dots, N-1$$

and

$$\bar{a}_N = \frac{1}{N} \sum_{s=0}^{N-1} f\left(\cos \frac{s\pi}{N}\right) \cos s\pi ,$$

where the double prime indicates that the first and last terms are to be factored by one-half. The accuracy of the coefficients is made sufficient by choosing N large enough, and the appropriate value of N depends on the rate of convergence of the series.

Putting

$$x_s = \cos \frac{s\pi}{N},$$

the expression for the coefficients takes the form

$$\bar{a}_r = \frac{2}{N} \sum_{s=0}^{N-1} f(x_s) T_r(x_s)$$

which is convenient for the computation. The factor 1/2 is required for \bar{a}_N .

A truncated Chebyshev series can be evaluated by a simple recurrence process due to Clenshaw. If

$$f(x) = \sum_{r=0}^{N-1} a_r T_r(x),$$

put

$$b_{N+2} = 0,$$

$$b_{N+1} = 0,$$

$$b_r = 2x b_{r+1} - b_{r+2} + a_r; \quad r = N, N-1, \dots, 0,$$

then

$$f(x) = \frac{1}{2} (b_0 - b_2).$$

This process can also be used to determine the coefficients, \bar{a}_r , above (see Reference 25).

If

$$\frac{d}{dx} f(x) = \sum_{r=0}^{\infty} a_r' T_r(x),$$

the integral formula gives

$$f(x) = \sum_{r=0}^{\infty} a_r T_r(x)$$

with

$$a_r' = \frac{1}{2r} (a_{r-1}' - a_{r+1}') , \quad r = 1, 2, 3, \dots ,$$

and a_0 is the arbitrary constant.

The derivative of the function can be evaluated using the coefficients a_r of the function itself by a combination of the recurrence relation

$$a_{N+1}' = 0 ,$$

$$a_N' = 0 ,$$

$$a_r' = 2(r+1) a_{r+1} + a_{r+2}' , \quad r = N-1, N-2, \dots, 0 ,$$

with the process given on page 4 for evaluating a Chebyshev series. This facilitates the computation of velocities.

If

$$f_1(x) = \sum_{r=0}^{\infty} a_r T_r(x) ,$$

and

$$f_2(x) = \sum_{s=0}^{\infty} b_s T_s(x) ,$$

then

$$f_1(x) f_2(x) = \sum_{k=0}^{\infty} c_k T_k(x) ,$$

with

$$c_0 = \frac{1}{2} a_0 b_0 + \sum_{r=1}^{\infty} a_r b_r ,$$

and

$$c_k = \frac{1}{2} \sum_{r=0}^{\infty} (a_r b_{r+k} + a_{r+k} b_r) + \frac{1}{2} \sum_{r=1}^{k-1} a_r b_{k-r} , \quad \text{for } k > 0 ,$$

from the product formula.

With

$$f(x) = \sum_{r=0}^{\infty} a_r T_r(x)$$

the values of the function and its derivative for $x = 0$ are given by

$$f(0) = \frac{1}{2} a_0 - a_2 + a_4 - a_6 + \dots$$

$$\left[\frac{d}{dx} f(x) \right]_{x=0} = a_1 - 3a_3 + 5a_5 - 7a_7 + \dots$$

These are all of the formulas which will be needed in the present application.

CHEBYSHEV EXPANSIONS OF TRIGONOMETRIC FUNCTIONS

In the classical form, the general perturbations are given as series consisting of secular terms, periodic terms, and mixed (Poisson) terms. It is of interest to see how functions of these types appear when expanded in Chebyshev series.

The independent variable, x , of the Chebyshev polynomials is taken to be a linear transformation of the time, t . A secular term x^k is represented exactly by a linear combination of the Chebyshev polynomials $T_r(x)$ for $r = 0, 1, 2, \dots, k$. The expansions of the periodic terms $\cos(px)$ and $\sin(px)$ are given below. A mixed term $x^k \cos(px)$ or $x^k \sin(px)$ will be represented by the product of the expression for x^k with the series for the periodic term. Thus the general characteristics of the Chebyshev expansion of a mixed term for k small will be similar to that for the periodic term. An examination of the Chebyshev expansions of cosine and sine functions gives an indication of the number of terms which will be required in the representation of the planetary perturbations in this form.

Put

$$\cos(px) = \sum_{r=0}^{\infty} C_r^{(p)} T_r(x)$$

and

$$\sin(px) = \sum_{r=0}^{\infty} S_r^{(p)} T_r(x)$$

where, as before, the prime indicates that the first term is to be factored by one-half. Since $\cos(px)$ is an even function and $\sin(px)$ is an odd function,

$$C_r^{(p)} = 0 \text{ for } r \text{ odd}$$

and

$$S_r^{(p)} = 0 \text{ for } r \text{ even}.$$

The non-zero coefficients are given by the definite integrals

$$C_r^{(p)} = \frac{2}{\pi} \int_0^\pi \cos r\theta \cos(p \cos \theta) d\theta \text{ for } r \text{ even}$$

and

$$S_r^{(p)} = \frac{2}{\pi} \int_0^\pi \cos r\theta \sin(p \cos \theta) d\theta \text{ for } r \text{ odd}.$$

These expressions are recognized to be the Bessel functions of integer order with a simple factor:

$$C_r^{(p)} = (-1)^{r/2} 2J_r(p) \text{ for } r \text{ even}$$

and

$$S_r^{(p)} = (-1)^{(r-1)/2} \cdot 2J_r(p) \text{ for } r \text{ odd}.$$

Thus the behavior of the coefficients in the expansions of the cosine and sine functions is related to the behavior of the Bessel functions of fixed argument and increasing integer order. These functions have been extensively investigated and tabulated. Considering $J_r(p)$ where r and p are positive and p is small (corresponding to terms of long period), the values of $J_r(p)$ decrease in magnitude rapidly as r increases. These cases present no difficulty. When p is large (corresponding to terms of short period), three cases are encountered corresponding to r/p less than, nearly equal to, or greater than unity. For the present purpose it is sufficient to notice that for $r < p$ the values of $J_r(p)$ oscillate about the value zero as r increases and for $r > p$ the values decrease rapidly in magnitude. The behavior of the coefficients is illustrated in Figure 1 for the case $p = 100$.

Figure 2 may be used to determine, for a given value of p , that value of k for which $|2J_r(p)|$ is less than 10^{-4} , 10^{-10} or 10^{-16} for all $r > k$. These results give a good

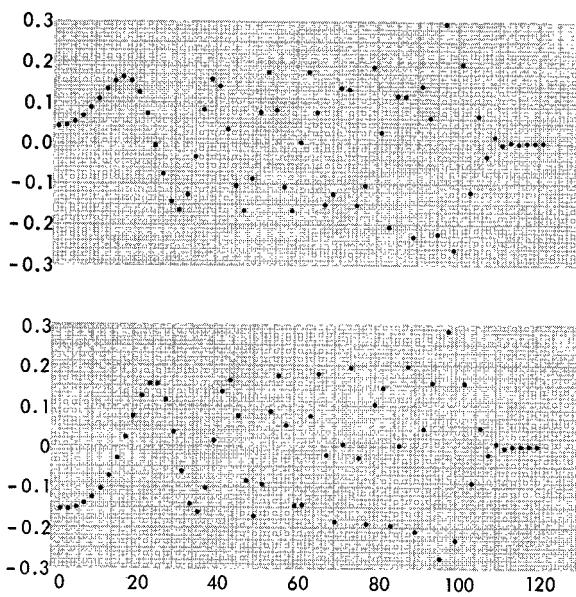


Figure 1—(a) – Coefficients for r even in the expansion of $\cos(100x)$. (b) – Coefficients for r odd in the expansion of $\sin(100x)$.

indication of the number of terms which will be required in the expansion of the planetary perturbations when the shortest period of the significant periodic terms is known in relation to the time interval over which the series are to be valid. If a time interval, Δt , is chosen and a term $\cos 2\pi t/\tau$, with period τ is considered, this term will be a linear combination of $\cos(px)$ and $\sin(px)$ with

$$p = \frac{\Delta t}{\tau} \pi .$$

For example, for a two century interval and a term whose period is 2π years, $p = 100$. From Figure 2

$$\log_{10} \left(\frac{k}{p} \right) = \begin{cases} 0.07 & \text{for } |2J_r(p)| < 10^{-4} \\ 0.135 & \text{for } |2J_r(p)| < 10^{-10} \\ 0.18 & \text{for } |2J_r(p)| < 10^{-16} \end{cases} \text{ for } r > k ,$$

and the resulting numbers of terms are 118, 137 and 152 respectively.

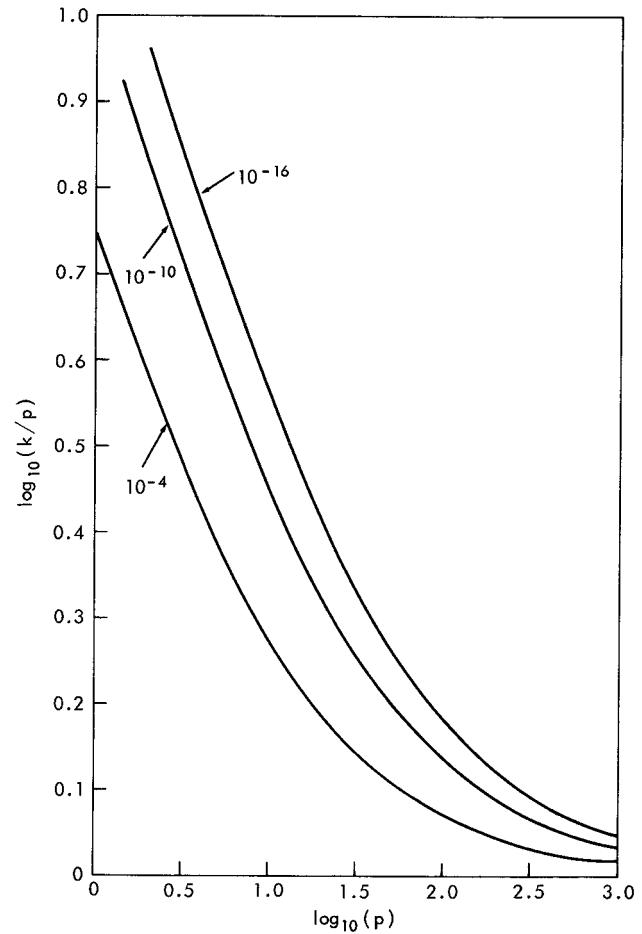


Figure 2—Values of $\log_{10}(k/p)$ for which $|2J_r(p)| < 10^{-4}$, 10^{-10} , and 10^{-16} for $r > k$.

EQUATIONS FOR THE PERTURBATIONS

The Chebyshev series may be used with nearly any decomposition of the perturbations. By computing perturbations of the coordinates the number of components can be held to three, although perturbations of the elements may be more descriptive. Musen's (Reference 12) method of perturbations of the rectangular coordinates is applied in the present case.

The position vector, \vec{r} , of a planet is given by

$$\vec{r} = (1 + \alpha) \vec{r}_0 + \beta \vec{w} + \gamma a \vec{R}$$

where α, β, γ are the components of the perturbations, \vec{r}_0 , is the position vector in the fixed reference ellipse, a is the semimajor axis of the reference ellipse, \vec{R} is the unit vector in the direction of the angular momentum of the motion in the reference ellipse and

$$\vec{w} = \frac{1}{n} \frac{d\vec{r}_0}{dt},$$

n being the mean motion in the reference ellipse and t the time.

The perturbations are computed from the formulas

$$\alpha = \sin \epsilon \int Q_1 d(nt) + (\cos \epsilon - e) \int Q_2 d(nt) + 2M_4$$

$$\beta = \int (M_4 - 2\alpha) d(nt)$$

$$\gamma = \sin \epsilon \int R_1 d(nt) + (\cos \epsilon - e) \int R_2 d(nt)$$

with

$$\vec{S}_1 = \vec{P} \cos \epsilon + \vec{Q} \frac{\sin \epsilon}{\sqrt{1-e^2}}$$

$$\vec{S}_2 = -\vec{P} \sin \epsilon + \vec{Q} \frac{\cos \epsilon - e}{\sqrt{1-e^2}}$$

$$M_1 = a^2 \vec{F} \cdot \vec{S}_1$$

$$M_2 = a^2 \vec{F} \cdot \vec{S}_2$$

$$M_3 = a^2 \vec{F} \cdot \vec{R} \frac{\vec{r}_0}{a}$$

$$M_4 = \int M_2 d(nt)$$

$$Q_1 = \frac{a}{r_0} [M_1 (\cos \epsilon - e) - 2M_2 \sin \epsilon]$$

$$Q_2 = \frac{a}{r_0} [-M_1 \sin \epsilon - 2M_2 \cos \epsilon]$$

$$R_1 = \frac{a}{r_0} M_3 (\cos \epsilon - e)$$

$$R_2 = -\frac{a}{r_0} M_3 \sin \epsilon$$

The quantities ϵ and e are the eccentric anomaly and the eccentricity respectively of the reference motion, and \vec{P}, \vec{Q} are the usual unit vectors of the fixed reference ellipse. The remaining undefined quantity is the disturbing force, \vec{F} , which is given by (see Reference 19, page 2)

$$\vec{F} = \left(-\frac{\vec{r}}{r^3} + \frac{\vec{r}}{r_0^3} - 3 \frac{\vec{r}_0 \cdot \vec{s}}{r_0^5} \vec{r}_0 \right) + \sum_{i=1}^N \frac{m_i}{M+m} \left(\frac{\vec{r}_i}{r_i^3} - \frac{\vec{r}_i}{r_0^3} \right).$$

It is not necessary in the present case to expand \vec{F} in powers of the perturbations, but an expansion of the solar terms may be desirable to prevent loss of accuracy in the numerical computation. The disturbing force depends on the actual position vectors, so a process of iteration is implied in computing the perturbations. It is understood that the contributions from the constants of integration are to be added to the above expressions for α, β and γ . The effects of general relativity will also be added as small corrections in the usual way. The usual method of introducing a fictitious anomaly under the integral signs is not convenient for the present method, so the quantities $\sin \epsilon$ and $\cos \epsilon - e$ have been brought outside as factors. There was some concern that numerical accuracy would be lost in this way, but tests show that this is not the case in the present program where the equivalent of sixteen figures are carried in the computation.

The integrands are expanded in Chebyshev series for a pre-determined time interval by the method of special values described on page 3. Corresponding series are obtained for the functions

$\cos \epsilon$ and $\sin \epsilon$. The series integrations and multiplications indicated in the computation of α , β and γ are then performed using the given formulas. If the results are to be valid in the time interval $[t_1, t_2]$, then the time, t , is transformed into the independent variable, x , of the Chebyshev functions by putting

$$t_0 = \frac{1}{2} (t_1 + t_2)$$

and

$$K = \frac{2}{t_2 - t_1}$$

so that

$$x = K(t - t_0) .$$

The relation

$$d(nt) = \frac{n}{K} dx$$

is used in performing the integrations and in evaluating the derivatives of the perturbations at the epoch.

The constants of integration are, in the present case, determined from the initial conditions. Let a prime signify differentiation with respect to nt , and let $A_0, B_0, C_0, A'_0, B'_0, C'_0$ be the numerical values at the epoch of the perturbations obtained by the formal integration. The constants z_1, z_2, \dots, z_6 are then computed using the predetermined initial conditions $\alpha_0, \beta_0, \gamma_0, \alpha'_0, \beta'_0, \gamma'_0$ as follows

$$z_3 = 2(\alpha_0 - A_0) + (\beta_0' - B_0')$$

$$z_1 = (\alpha_0 - A_0 - 2z_3) \frac{a}{r_0} \cos \epsilon_0 - (\alpha'_0 - A'_0) \sin \epsilon_0$$

$$z_2 = (\alpha_0 - A_0 - 2z_3) \frac{a}{r_0} \sin \epsilon_0 + (\alpha'_0 - A'_0) (\cos \epsilon_0 - e)$$

$$z_4 = (\beta_0 - B_0) + \left[(2 - e^2) \sin \epsilon_0 - \frac{e}{2} \sin 2\epsilon_0 \right] z_1 + \left[-2 \cos \epsilon_0 + \frac{e}{2} \cos 2\epsilon_0 \right] z_2$$

$$z_5 = (\gamma_0 - C_0) \frac{a}{r_0} \cos \epsilon_0 - (\gamma'_0 - C'_0) \sin \epsilon_0$$

$$z_6 = (\gamma_0 - C_0) \frac{a}{r_0} \sin \epsilon_0 + (\gamma'_0 - C'_0) (\cos \epsilon_0 - e) .$$

The contributions to the series for the perturbations are now added

$$\alpha = A + 2z_3 + z_1(\cos \epsilon - e) + z_2 \sin \epsilon$$

$$\beta = B + z_4 + 3(ez_1 - z_3)nt + \left[-(2 - e^2) \sin \epsilon + \frac{e}{2} \sin 2\epsilon \right] z_1 + \left[2 \cos \epsilon - \frac{e}{2} \cos 2\epsilon \right] z_2$$

$$\gamma = C + z_5(\cos \epsilon - e) + z_6 \sin \epsilon ,$$

where A, B, C represent the series obtained by the formal integration process. These formulas were extracted from Musen's work (Reference 12) with slight modifications of the notation.

All of the operations indicated in this section are performed by the program CHBY described and listed in Appendix A.

COMPARISON WITH TABULATED COORDINATES

For purposes of checking or determination of the constants it may be desired to compare the computed perturbations with available tabulated coordinates. In making the comparison it is necessary to allow for an elliptic correction. The method of Eckert and Brouwer (Reference 29) is used for this purpose. Let the tabulated position vectors be denoted by \vec{r}_T and represent them in the form $\vec{r}_T = \vec{r}_0 + \delta \vec{r}_T$,

$$\delta \vec{r}_T = \alpha_T \vec{r}_0 + \beta_T \vec{w} + \gamma_T a \vec{R}$$

by putting

$$\delta_1 = \delta \vec{r}_T \cdot \vec{P}$$

$$\delta_2 = \delta \vec{r}_T \cdot \vec{Q}$$

$$\delta_3 = \delta \vec{r}_T \cdot \vec{R}$$

and

$$\alpha_T = \frac{a}{r_0} \left(\frac{1}{a} \delta_1 \cos \epsilon + \frac{1}{a\sqrt{1-e^2}} \delta_2 \sin \epsilon \right)$$

$$\beta_T = -\frac{1}{a} \delta_1 \sin \epsilon + \frac{1}{a\sqrt{1-e^2}} \delta_2 (\cos \epsilon - e)$$

$$\gamma_T = \frac{1}{a} \delta_3 .$$

Similarly the position vector computed from the series has the form

$$\vec{r} \approx \vec{r}_0 + \delta\vec{r}$$

with

$$\delta\vec{r} = \alpha\vec{r}_0 + \beta\vec{w} + \gamma a\vec{R}.$$

A set of constants of integration z_1, z_2, \dots, z_6 are to be determined such that when their contributions are added to α, β and γ as in the previous section, the differences $\alpha - \alpha_T, \beta - \beta_T$ and $\gamma - \gamma_T$ are minimized in the sense of least squares over the tabulated dates. The comparisons are given equal weights. At each tabular date there are three condition equations, one for each component of the perturbations.

The α equation is

$$a_1 z_1 + a_2 z_2 + a_3 z_3 = a_0$$

where

$$a_0 = \alpha - \alpha_T$$

$$a_1 = \cos \epsilon - e$$

$$a_2 = \sin \epsilon$$

$$a_3 = 2.$$

The β equation is

$$a_1 z_1 + a_2 z_2 + a_3 z_3 + a_4 z_4 = a_0$$

where

$$a_0 = \beta - \beta_T$$

$$a_1 = 3e(nt) - (2 - e^2) \sin \epsilon + \frac{e}{2} \sin 2\epsilon$$

$$a_2 = 2 \cos \epsilon - \frac{e}{2} \cos 2\epsilon$$

$$a_3 = -3(nt)$$

$$a_4 = 1.$$

The γ equation is

$$a_5 z_5 + a_6 z_6 = a_0$$

where

$$a_0 = \gamma - \gamma_T$$

$$a_5 = \cos \epsilon - e$$

$$a_6 = \sin \epsilon .$$

To solve these equations in the sense of least squares, put

$$c_j = \sum_k a_0 a_j$$

$$b_{i,j} = \sum_k a_i a_j \quad \text{for } i, j = 1, 2, 3, \dots, 6$$

the index k extending over the tabular dates, each of the three sets of a_i being taken at each date. Let B be the matrix $\{b_{i,j}\}$, then

$$\begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_6 \end{pmatrix} = B^{-1} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_6 \end{pmatrix}$$

where B^{-1} is the inverse of the symmetric matrix B . The contributions of these integration constants are added to α , β and γ to give the reduced residuals. The effects of the integration constants may also be taken into account by an adjustment of the elliptic reference elements using the formulas

$$\frac{\delta a}{a} = 2z_3 - 2ez_1$$

$$\delta e = -(1 - e^2) z_1$$

$$\delta g_0 = z_4 + \left(\frac{1}{e} + \frac{e}{2} \right) z_2$$

$$\delta i = \delta p \cos \omega - \delta q \sin \omega$$

$$\sin i \delta\Omega = \delta p \sin \omega + \delta q \cos \omega$$

$$\delta\omega + \cos i \delta\Omega = \delta r$$

where

$$\delta p = \frac{1}{\sqrt{1-e^2}} z_6$$

$$\delta q = -z_5$$

$$\delta r = -\frac{\sqrt{1-e^2}}{e} z_2$$

(see Reference 29). Care must be taken when e is very small. The relationships between the z_i and the Eckert-Brouwer parameters are

$$z_1 = -\frac{1}{2(1-e^2)} \xi_6$$

$$z_2 = -\frac{1}{2\sqrt{1-e^2}} \xi_4^*$$

$$z_3 = \frac{1}{20} \xi_5 - \frac{e}{2(1-e^2)} \xi_6$$

$$z_4 = \xi_1^* + \frac{2+e^2-2\sqrt{1-e^2}}{4e\sqrt{1-e^2}} \xi_4^*$$

$$z_5 = -\xi_3^*$$

$$z_6 = \sqrt{1-e^2} \xi_2^* .$$

If the elliptic elements are held fixed, the effects of the constants of integration may be reflected in a modification of the initial conditions used in the perturbation program by adding to α_0 , β_0 , γ_0 the contributions as given on page 12, by letting A'_0 , B'_0 , C'_0 be the previous values

of the derivatives and putting

$$\alpha_0' = A_0' + \frac{a}{r_0} (-z_1 \sin \epsilon_0 + z_2 \cos \epsilon_0)$$

$$\beta_0' = B_0' + 3(ez_1 - z_3) + \frac{a}{r_0} \left\{ \left[-(2 - e^2) \cos \epsilon_0 + e \cos 2\epsilon_0 \right] z_1 + \left[-2 \sin \epsilon_0 + e \sin 2\epsilon_0 \right] z_2 \right\}$$

$$\gamma_0' = C_0' + \frac{a}{r_0} (-z_5 \sin \epsilon_0 + z_6 \cos \epsilon_0).$$

The operations indicated in this section were used to compare the series for the five outer planets with the coordinates given in Reference 30.

It should be mentioned that series corresponding to those computed as in the previous section can be obtained by numerical analysis of the tabular values α_T , β_T , γ_T using interpolation to obtain the special values. This process was used to determine the practical value of using Chebyshev series in the present context and to provide a basis for code checking the CHBY program. The planetary coordinate series (and, with slight modification, the lunar coordinates) produced by this method are especially convenient for use in programs for interplanetary trajectories, lunar expeditions, etc. For the reasonably short time intervals involved in such programs, the coordinates can be computed from the series much more efficiently than they can be read from a magnetic tape or other medium. If the velocities are needed they may be computed from the same series using the recurrence process for the derivative of a Chebyshev series as given on page 5.

APPLICATIONS

The Chebyshev method is applied here to the mutual attractions of the five outer planets with an augmented mass of the sun using the same constants as those used by Eckert, Brouwer and Clemence (Reference 30) for the numerical integration of the same system of equations. The chosen time interval is from Julian Date (JD) 2378400.5 (September 26.5, 1799) through JD 2452000.5 (April 1.0, 2001) with the central epoch JD 2415200.5 (June 29.5, 1900), each time being a standard 400-day date.

The Keplerian elements of the planets were determined by a least squares fit to the coordinates given in Reference 30 at the 80-day standard dates from JD 2400000.5 through 2430400.5. They should correspond roughly to mean elements with the affects of all long period terms included. The exact values which were used are given in Table 1. The Eulerian angles are referred to the ecliptic and mean equinox of 1950.0.

Table 1
Reference Elements of the Planets.

Planet	Semi-Major Axis, a , in a.u.	Eccentricity, e	Mean Motion in Degrees/Day, n	Longitude of Ascending Node, Ω	Argument of Perihelion, ω	Inclination, i	Mean Anomaly at Epoch, g_0
Jupiter	5.202 9737	0.04852 5422	0°083087 39726 97137	99°848 097	274°169 895	1°308 122	239°954 874
Saturn	9.537 1950	0.05417 9408	0.033468 51033 67896	113.382 301	338.991 032	2.486 946	180.374 080
Uranus	19.184 3420	0.04732 8819	0.011729 90526 83551	73.745 938	96.312 192	0.773 735	76.119 648
Neptune	30.072 9814	0.00885 2234	0.005976 56828 47765	131.236 967	275.039 422	1.774 465	40.524 435
Pluto	39.487 0157	0.24826 6637	0.003972 13962 76679	109.634 196	113.719 012	17.143 503	230.365 375

The mean motions are derived from the semi-major axes using Kepler's Law with the augmented mass of the sun. There is no advantage to be had from using a corrected value of the semi-major axis because the perturbations are not separated into various orders. The adopted values of the masses are the same as those in Reference 30. The reciprocal values are given in Table 2.

Table 2
Reciprocal Mass for Planets.

Planet	Reciprocal Mass, $1/m$
Sun	1
Jupiter	1047.355
Saturn	3501.6
Uranus	22869.
Neptune	19314.
Pluto	360000.

The sum of the masses of the inner planets is taken as $5.97682 \cdot 10^{-6}$. This value is added to the mass of the sun for applying Kepler's Law relating the semi-major axis and the mean motion and for evaluating the disturbing force \vec{F} from the formula given on page 10.

In the past it has been the custom to give the details of the intermediate computations for planetary theories. With modern methods a much more useful approach is to publish the program which was used. In this way, the reader may duplicate the computations if he wishes, he may check the program for any possible errors, and he may apply the method to any other cases in which he may be interested. In fact, the program represents a much more valuable end result

than the specific examples. It is now possible with a few minutes of computer time to obtain planetary theories for any cases of interest to a very high degree of accuracy.

The coefficients in the series for the perturbations are given in Appendix B. The constants of integration have been adjusted to give the best fit to the coordinates given in Reference 30. To establish the formal accuracy of the results, they have been compared with the tabulated coordinates at the standard 400-day dates from JD 2378400.5 through JD 2452000.5 using the method described in the previous section. The results of the comparisons are given in Appendix C. In computing the perturbations, the iteration process was continued until the residuals were in all cases less than 0".001. Residuals of this magnitude are expected to result from rounding the coefficients in the series at the tenth decimal. The purpose of this test was to reveal any errors in the program. The actual positions of the planets are known with much less accuracy, and the additional terms from the inner planets and the relativistic effects are much larger than these residuals.

It might seem logical now to include the remaining perturbations from the inner planets and the relativistic corrections. Since the contributions from the inner planets consist primarily of small terms of short period, I have preferred to apply the usual trigonometric form for their elaboration. The results are given in Appendix D.

The reader may be interested to know how many iterations were required in the solution since this relates to the convergence along powers of the disturbing masses. To provide an answer, the perturbations of Jupiter were compared with the tabulated coordinates at each iteration. The results are given in Table 3.

Table 3
Residuals for Jupiter at Each Iteration.

	Maximum Residuals			rms Residuals			
	$\alpha \cdot 10^{10}$	$\beta \cdot 10^{10}$	$\gamma \cdot 10^{10}$	$\alpha \cdot 10^{10}$	$\beta \cdot 10^{10}$	$\gamma \cdot 10^{10}$	
Elliptic Elements	9162977	26118663	911236	4292639	9844551	357793	
Iteration							
1	61397	214572	2682	22165	95756	1041	
2	1122	18001	21	436	7421	6	
3	44	342	12	14	205	4	
4	28	33	11	8	14	4	

The entries in the first line relate to the total perturbations after removal of a small elliptic correction. The residuals are less than 0".01 after the third iteration and less than 0".001 after the fourth.

It is of interest to know the magnitude of the effects on the motion of a planet due to the perturbations of the disturbing planets. This relates, for example, to the importance of the second order terms in the motion of Saturn due to the perturbations of Uranus. For this purpose the perturbations of Saturn have been computed with the coordinates of one disturbing planet given by the reference elements alone. The process was repeated for each disturbing planet, and the results were compared with the tabulated coordinates. Allowing for an elliptic correction, the results are shown in Table 4.

Table 4

Residuals for Saturn with Disturbing Planets in Elliptic Orbits.

Planet in Elliptic Orbit	Maximum Residuals			rms Residuals		
	$\alpha \cdot 10^{10}$	$\beta \cdot 10^{10}$	$\gamma \cdot 10^{10}$	$\alpha \cdot 10^{10}$	$\beta \cdot 10^{10}$	$\gamma \cdot 10^{10}$
Jupiter	112014	290201	4719	47089	102044	1898
Uranus	7914	6578	437	1406	2338	116
Neptune	574	561	14	131	209	5
Pluto	37	107	16	9	20	5

CONCLUSION

The practical value of using Chebyshev series in solving the equations of planetary motion has been demonstrated. The accurate solutions for the motions of the five outer planets serve as a basis for computing perturbations of the minor planets. The small contributions from the inner planets are obtained using the first order expansion in multiples of the mean anomalies.

For more eccentric orbits the convergence of the series may be improved by letting the independent variable, x , of the Chebyshev polynomials be a linear function of the eccentric anomaly of the disturbed planet rather than a linear function of the time. This approach would require only a slight modification of the program, and the computation time would be reduced because of the elimination of many solutions of the Kepler equation.

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Appendix A

Program CHTY

The FORTRAN program CHTY together with its subroutines performs the series expansions and the formal integrations, determines the constants of integration to fit the initial conditions and adjusts the coefficients accordingly. The final values of the coefficients are printed and punched on cards. The first data card contains 72 BCD characters used as a title to identify the run.

The next data card contains the control indices used in the program. They are:

- N1+1 the number of special values used in the expansions
- NTRM the number of terms computed in the series
- ITRM the number of iterations to be made in the run
- IPRINT the print control, normally put equal to 1. For larger values intermediate results are printed
- IPUNCH the punch control. For IPUNCH = 1 cards are not punched. For IPUNCH = 2 cards are punched

The next data card contains the Julian Date of the epoch (TEPOCH), the time interval in days for which the theory is valid (DTCHEB), and the smallest number of units in the last coefficients to be printed and punched (EPS1). In the examples

TEPOCH = 2415200.5

DTCHEB = 73600.0

EPS1 = 0.5.

The next data card contains the Julian Date of the epoch (PEPOCH), and the time interval in days (PTCHEB) for the disturbing planets. The time interval of validity of the theories for the disturbing planets must contain the corresponding interval for the disturbed planet. In the examples, the intervals are the same, but the results may be used, for example, to determine the motion of a minor planet between 1900 and 2000.

The next two data cards contain double precision values of the reference elements:

SA semi-major axis in astronomical units

SE eccentricity

SO2	argument of perihelion in degrees
SI2	inclination in degrees
CO2	longitude of the ascending node in degrees
SM	mass of the disturbed planet in units of the sun's mass
SGZ2	mean anomaly at the epoch in degrees
SMINNR	mass of inner planets to be added to the sun

The elements define the reference orbit. They are usually referred to the ecliptic and mean equinox of 1950, but it is only necessary that the coordinate system be the same as that used for the disturbing planets. The vectorial constants to the equator are computed from the elements using Newcomb's value of the obliquity for 1950.

The next two data cards contain the constants of integration ZK(I), I = 1, 6 which may have been obtained from a previous comparison with observations or tabulated coordinates. They enter the computation of the position vectors used for evaluating the disturbing force.

The next two data cards contain the initial conditions used in determining the new constants of integration. The initial conditions are defined in terms of the values of the perturbations, α , β and γ and their derivatives with respect to nt at the epoch.

The next data cards contain the previous values of the coefficients in the series for the perturbations. The end of this deck is followed by a card with the digits 1001 punched in columns 5 through 8. If there are no previous coefficients, the single 1001 card must be present.

The remaining data cards are read by the FFD subroutine described below. The operations being performed at various stages of the program are described by the comment cards. The double precision coefficients of the Chebyshev series are stored in the two-dimensional x array. The second subscript designates the series, and the first subscript relates to the individual coefficients of the series. To make full use of the computer memory, some areas of storage are shared by different series in various parts of the program.

The CHBY program uses five subroutines which will now be described.

PQRDP2. This subroutine computes the vectorial constants to the equator from the Eulerian angles referred to the ecliptic.

FFD. This is the main subroutine of the program. Its function is to evaluate the disturbing force at each of the collocation points. The first time the subroutine is called, it reads the

remainder of the data. There is one set of data for each disturbing planet. Each set consists of the following:

1. Title card containing 72 BCD characters.
2. Two data cards containing the elliptic elements in the same format as for the disturbed planet previously described.
3. Cards containing the coefficients in the series for the perturbations of the disturbing planet. The end of each deck is followed by a card with the digits 1001 punched in columns 5 through 8, except that the last set has the digits 1002 on the final card.

The steps in the computation are straightforward. The solar terms of the disturbing force are expanded in powers of the perturbations.*

COEFF. This subroutine adds the contributions of the special values to the coefficients in the series expansions.

MLTPLY. This subroutine performs the Chebyshev series multiplications.

NTGRT. This subroutine performs the Chebyshev series integrations.

Care was taken in the programming, and various tests were made for the detection of errors. However, the possibility remains that there are errors which did not make any significant difference in the given examples, but which could show up in a more extreme case.

*Carpenter, L., "Computation of General Planetary Perturbations, Part III—An Expansion of the Distributing Force," NASA Technical Note D-3078, 1965.

Table A-1
Listing of CHBY and Subroutines.

```

C  CARPENTER, CHBY PROGRAM, COMPUTES PLANETARY PERTURBATIONS (ALPHA,
C  BETA, GAMMA) IN CHEBYSHEV SERIES
      DOUBLE PRECISION EKK,XJ,V,SA,SB,SE,SN,SGZ,SM,EP,P,Q,R,EK,RAD,
      1PI,TWUPI,EP2,SO2,S12,C02,SGZ2,SMINNR,SO,SI,CO,SN2,A,B,C,THET,DLL,
      2E,CUSC,SINC,ESE,ECE,BK,DCOS,DSIN,COS2C,SIN2C,X,VAL,VALZ,FAC,ZK,
      3TWUSE,CN1,CN2,CN3,CN4,DN1,DN2,CNN,SAB,TRATIO,DXZR,DSQRT
      COMMON X,EKK,NTRM,N1,XJ,V,SA,SB,SE,SN,SGZ,SM,EP,P,Q,R,IPRINT,NTIM
      COMMON TRATIO,DXZR,ZK
      DIMENSION X( 701,8 ),AA(6),ZK(6),          V(5),ITITLE(12)
      DIMENSION A(3),B(3),C(3),P(3),Q(3),R(3),VAL(3,2),VALZ(3,2)
1 FORMAT (4F15.7)
2 FORMAT (1H08X1HX17X4HTHET41X1HV/1P7D18.10)
3 FORMAT (80(1H3))
5 FORMAT (I8,6P6F12.4)
7 FORMAT (26H0 VALUE OF SERIES AT EPOCH/ 11X1HA19X1HB19X
71   1HC16X7HA PRIME13X7HB PRIME13X7HC PRIME//)
8 FORMAT (1P6D20.10)
9 FORMAT (/2(7X1HK1X11HALPHA*1.E6 2X10HBETA*1.E6 1X11HGAMMA*1.E6 )//
91)
1J FORMAT (2(I8,6P3F12.4))
12 FORMAT (10H CARPENTER)
13 FORMAT (22H0INTEGRATION CONSTANTS/1H0,9X2HZK,I1,5(17X2HZK,I1))
15 FORMAT (918).
17 FORMAT (8H0 TEPUCH4X6HDTCHEB4X6HPEPOCH4X6HPTCHEB5X4HEPSI/5F10.1)
22 FORMAT (17H0TEST ZERO VALUES/1P6D20.12)
25 FORMAT (5H0 K55X2HN113X2HN213X2HN312X4HCOSE11X4HSINE//)
26 FORMAT (I5,48X,1P5E15.7)
29 FORMAT (26H0 INITIAL CONDITIONS / 11X1HA19X1HB19X
291  1HC16X7HA PRIME13X7HB PRIME13X7HC PRIME//)
32 FORMAT (7H0VALID ,F10.1,6H THRU ,F10.1)
33 FORMAT (4D18.11)
34 FORMAT (I8,0P3F12.0,10X,1P3E15.7)
36 FORMAT (48HJ N1 NTRM ITRM IPRINT IPUNCH )
37 FORMAT (28H0REFERENCE ELLIPTIC ELEMENTS/
371   1H013X2HSA23X2HSE22X3HS0222X3HS12/1P4D25.15)
38 FORMAT (1H012X3HCO223X2HSM22X4HSGZ220X6HSMINNR/1P4D25.15)
39 FORMAT (1H013X2HSB22X3HSN2/1P2D25.15)
41 FORMAT(12A6)
43 FORMAT(1H1)
52 FORMAT (64H0VECTORIAL CONSTANTS REFERRED TO EQUATOR AND MEAN EQUIN
521OX 1950.0/
522   1H010X1HP19X1HQ19X1HR19X1HA19X1HB19X1HC/(1P6D20.12))
      PI=3.141592653589793
      TWUPI=2.*PI
      RAD=TWUPI/360.
C
C  EK IS THE GAUSSIAN CONSTANT
C
C  EK=0.01720209895D+00
C
C  EP2 IS NEWCOMB OBLIQUITY FOR JAN 0 1950, 23DEG 26MIN 44.84SEC
C
C  EP2=23.44578868888889
      EP=EP2*RAD
C
C  ENTRY POINT FOR SUCCESSIVE CASES
C
55 CONTINUE
      DO 59 I=6,8
      DO 59 J=1, 701
59 X (J,I)=0.

```

TABLE A-1 (CONTINUED)

```

READ (5,41) ITITLE
C
C ITITLE IS 72 BCD CHARACTERS USED AS A TITLE FOR THE RUN
C
C     READ (5,15) N1,NTRM,ITRM,IPRINT,IPUNCH
C
C N1+1      NUMBER OF SPECIAL VALUES
C NTRM      NUMBER OF COEFFICIENTS (NOT TO EXCEED 701)
C ITRM      NUMBER OF ITERATIONS FOR THIS RUN
C IPRINT     =1 PRINT RESULTS ONLY
C             =2 PRINT SPECIAL VALUES AND EXPANSION COEFFICIENTS
C             =3 PRINT TEST QUANTITIES IN FFD SUBROUTINE
C IPUNCH    =1 DO NOT PUNCH COEFFICIENTS ON CARDS
C             =2 PUNCH COEFFICIENTS
C
C     READ (5,1) TEPOCH,DTCHEB, EPS1
C
C TEPOCH    JULIAN DATE OF THE EPOCH
C DTCHEB    TIME INTERVAL IN DAYS FOR VALIDITY OF THEORY
C EPS1      CRITERION FOR TRUNCATING COMPUTED SERIES (AFTER STATEMENT 410)
C
C     READ (5,1) PEPOCH,PTCHEB
C
C PEPOCH, PTCHEB ARE EPOCH AND TIME INTERVAL FOR DISTURBING PLANETS
C
C ASSIGN 570 TO NPUNCH
C IF (IPUNCH-1)62,62,60
60 PUNCH 12
PUNCH 3
PUNCH 3
PUNCH 3
IF (ITRM-1) 61,61,62
61 ASSIGN 568 TO NPUNCH
62 CONTINUE
READ (5,33) SA,SE,SU2,SI2,CO2,SM,SGZ2,SMINNR
C
C NOTATION FOR ELEMENTS OF DISTURBED PLANET
C SA        SEMI-MAJOR AXIS IN ASTRONOMICAL UNITS
C SE        ECCENTRICITY
C SU2       ARGUMENT OF PERIHELION (DEGREES)
C SI        INCLINATION (DEGREES)
C CO2       LONGITUDE OF ASCENDING NODE (DEGREES)
C SM        MASS OF DISTURBED PLANET IN UNITS OF SOLAR MASS
C SGZ2      MEAN ANOMALY AT EPOCH (DEGREES)
C SMINNR    MASS OF INNER PLANETS (TO BE ADDED TO SOLAR MASS)
C
C     READ (5,33)ZK
C
C ZK ARE INTEGRATION CONSTANTS USED FOR COMPUTING SPECIAL VALUES
C
SO=SO2*RAD
SI=SI2*RAD
CO=CO2*RAD
SGZ=SGZ2*RAD
SN=EK*DSQRT(1.+SMINNR+SM)/SA**1.5
EKK=.5*SN*DTCHEB
TRATIO=DTCHEB/PTCHEB
DXZR=(TEPOCH-PEPOCH)*2./PTCHEB
SN2=SN/RAD
SB=SA*DSQRT(1.-SE**2)
SAB=SA/SB

```

TABLE A-1 (CONTINUED)

```

      WRITE (6,43)
      WRITE (6,41)ITITLE
      WRITE (6,36)
      WRITE (6,15) N1,NTRM,ITRM,IPRINT,IPUNCH
      WRITE (6,37) SA,SE,SO2,SI2
      WRITE (6,38) CO2,SM,SGZ2,SMINNR
      WRITE (6,39) SB,SN2
      WRITE (6,17)TEPOCH,DTCHEB,PEPOCH,PTCHEB,EPS1
      XJ1=TEPOCH-DTCHEB/2.
      XJ2=TEPOCH+DTCHEB/2.
      WRITE (6,32) XJ1,XJ2
C
C  VALZ ARE THE INITIAL CONDITIONS
C
      READ (5,33) ((VALZ(I,J),I=1,3),J=1,2)
      WRITE (6,29)
      WRITE (6,8) ((VALZ(I,J),I=1,3),J=1,2)
      WRITE (6,13)(I,I=1,6)
      WRITE (6,8)ZK
C
      CALL PQRDP2(S0,SI,CO,P,Q,R,EP)          PQRDP2
      DO 65 I=1,3
      A(I)=SA*P(I)
      B(I)=SB*Q(I)
      65 C(I)=SA*R(I)
      WRITE (6,52) (P(I),Q(I),R(I),A(I),B(I),C(I),I=1,3)
C
C  READ INPUT PERTURBATIONS
C
      NTIM=0
      WRITE (6,9)
      70 READ (5,5) K,(AA(I),I=1,6)
C
C  EACH CARD CONTAINS THE COEFFICIENTS OF T SUB K OF X AND T SUB K+1
C  OF X FROM THE PREVIOUS ESTIMATES OF ALPHA, BETA, GAMMA
C
      KP=K+1
      WRITE (6,10)K,(AA(I),I=1,3),KP,(AA(I),I=4,6)
      IF (K-1001) 80,92,92
      80 K1=K+1
      KP=K+2
      IF (K1-700) 82,82,70
      82 CONTINUE
      NTIM=KP
      DO 90 I=1,3
      J=I+3
      X(K1,I+5)=AA(I)
      X(KP,I+5)=AA(J)
      90 CONTINUE
      GO TO 70
      92 CONTINUE
      ITYPE=1
C
C  CALL FFD TO READ DISTURBING PLANET DATA          FFD
C
      CALL FFD (ITYPE)
      ITTRM=1
C
C  ENTRY POINT FOR SUCCESSIVE ITERATIONS
C
      94 ITYPE=2

```

TABLE A-1 (CONTINUED)

```

DO 96 I=1,5
DO 96 J=1, 701
96 X (J,I)=0.

C THE FOLLOWING OPERATIONS THRU STATEMENT 230 GIVE THE CHEBYSHEV
C EXPANSIONS OF THE DISTURBING FORCES. THE EXPANDED QUANTITIES ARE
C      V(1)          V(2)          V(3)          V(4)          V(5)
C      M1*ADR        2.*M2*ADR    M3*ADR       COS(E)        SIN(E)
C
C WHERE M1, M2, M3 ARE THE SUBSCRIPTED QUANTITIES FROM THE EQUATIONS
C FOR THE PERTURBATIONS, ADR IS THE FACTOR SMALL A/ SMALL R SUB ZERO,
C AND E IS THE ECCENTRIC ANOMALY IN THE REFERENCE ELLIPSE
C
C      XJ=-1.D+0
C
C CALL FFD FOR FIRST SET OF SPECIAL VALUES          FFD
C
C      CALL FFD (ITYPE)
C      THET=PI
C      IF (IPRINT-2)110,100,100
100  CONTINUE
      WRITE (6,2)XJ ,THET ,(V(I),I=1,5)
110  CONTINUE
      DO 200 I=1,5
      V(I)=.5D+0*V(I)
200  CONTINUE

C COF : ADDS CONTRIBUTIONS OF SPECIAL VALUES TO SERIES COEFFICIENTS   COEFF
C
C      CALL COEFF (XJ,V)
C      EN1=N1
C      DLL=PI/EN1
C      DO 210 K=2,N1
C      THET=THET-DLL
C      XJ=DCOS(THET)

C CALL FFD FOR INTERIOR POINTS          FFD
C
C      CALL FFD (ITYPE)
C
C      CALL COEFF (XJ,V)
C      IF (IPRINT-2)210,202,202
202  WRITE (6,2)XJ ,THET ,(V(I),I=1,5)
210  CONTINUE
      XJ=+1.D+0

C CALL FFD FOR LAST SET OF SPECIAL VALUES          FFD
C
C      CALL FFD (ITYPE)
C      IF (IPRINT-2)214,212,212
212  CONTINUE
      THET=0.
      WRITE (6,2)XJ ,THET ,(V(I),I=1,5)
214  CONTINUE
      DO 220 I=1,5
      V(I)=.5D+0*V(I)
220  CONTINUE

C      CALL COEFF (XJ,V)
C      FAC=2./EN1
C      N2=NTRM

```

TABLE A-1 (CONTINUED)

```

      DO 230 K=1,NTRM
      DO 230 I=1,5
      X(K,I)=X(K,I)*FAC
230 CONTINUE
      IF (IPRINT-2)364,240,240
240 CONTINUE
      WRITE (6,43)
      WRITE (6,41) ITITLE
      WRITE (6,25)
      DO 340 K=1,NTRM
      KP=K-1
      WRITE (6,26) KP,(X(K,I),I=1,5)
340 CONTINUE
C
C TEST ZERO VALUES OF X(K,1) THRU X(K,5)
C
      DO 362 I=1,5
      ZK(I)=.5*X(1,I)
      BK=1.
      DO 362 K=3,N2,2
      BK=-BK
      ZK(I)=ZK(I)+BK*X(K,I)
362 CONTINUE
      WRITE (6,22)(ZK(I),I=1,5)
C
C PERFORM SERIES INTEGRATIONS (THRU STATEMENT 410). THE PREVIOUS
C VALUES OF THE PERTURBATION COEFFICIENTS ARE NO LONGER NEEDED. THE NEW
C SERIES OCCUPY THE OLD LOCATIONS (X(K,6), X(K,7), X(K,8))
C
      364 TWOSE=2.*SE
      X(1,4)=X(1,4)-TWOSE
C
C MLTPLY (I1,I2,I3) CAUSES THE SERIES X(K,I1) AND X(K,I2) TO BE
C MULTIPLIED AND THE PRODUCT TO BE STORED IN X(K,I3)
C
      CALL MLTPLY (4,1,8)
      X(1,4)=X(1,4)+TWOSE
      CALL MLTPLY (5,2,7)
      DO 370 K=1,N2
      X(K,8)=X(K,8)-X(K,7)
370 CONTINUE
C
C NTGRT (I1,I2) CAUSES THE SERIES X(K,I1) TO BE INTEGRATED WITH RESPECT
C TO NT AND THE INTEGRAL TO BE STORED IN X(K,I2)
C
      CALL NTGRT (8,7)
      CALL MLTPLY (5,7,6)
      CALL MLTPLY (5,1,8)
      CALL MLTPLY (4,2,7)
      DO 380 K=1,N2
      X(K,8)=-X(K,8)-X(K,7)
380 CONTINUE
      CALL NTGRT (8,7)
      X(1,4)=X(1,4)-TWOSE
      CALL MLTPLY (4,7,8)
      X(1,4)=X(1,4)+TWOSE
      DO 390 K=1,N2
      X(K,6)=X(K,6)+X(K,8)
      X(K,1)=-SE*X(K,4)
390 CONTINUE
      X(1,1)=2.+X(1,1)

```

TABLE A-1 (CONTINUED)

```

CALL MLTPLY (1,2,7)
CALL NTGRT (7,8)
DO 400 K=1,N2
X(K,6)=X(K,6)+X(K,8)
X(K,8)=.5*X(K,8)-2.*X(K,6)
400 CONTINUE
C
C ALPHA IS NOW IN X(K,6)
CALL NTGRT (8,7)
C
C BETA IS NOW IN X(K,7)
X(1,4)=X(1,4)-TWOSE
CALL MLTPLY (3,4,1)
CALL NTGRT (1,2)
CALL MLTPLY (2,5,8)
CALL MLTPLY (3,5,1)
CALL NTGRT (1,2)
CALL MLTPLY (2,4,1)
DO 410 K=1,N2
X(K,8)=X(K,8)-X(K,1)
410 CONTINUE
C
C GAMMA IS NOW IN X(K,8)
C
C ESTABLISH TRUNCATION POINT, NTIM, FROM MAGNITUDES OF COEFFICIENTS
C
EPTST=EPS1/1.E10
NTIM=NTRM+1
DO 414 K=1,NTRM
NTIM=NTIM-1
DO 412 I=1,3
IF (ABS(X(NTIM,I+5))-EPTST) 412,416,416
412 CONTINUE
414 CONTINUE
416 CONTINUE
      WRITE (6,43)
      WRITE (6,41) ITITLE
C
C PRINT COEFFICIENTS
C
418 CONTINUE
      WRITE (6,9)
      DO 460 K=1,NTIM,2
K1=K-1
L=K+1
      DO 430 I=1,3
J=I+3
AA(I)=X(K,I+5)
AA(J)=X(L,I+5)
430 CONTINUE
      WRITE (6,10) K1,(AA(I),I=1,3),K,(AA(I),I=4,6)
460 CONTINUE
C
C EVALUATE SERIES AT EPOCH
C
      DO 500 I=1,3
      DO 500 J=1,2
VAL(I,J)=0.
500 CONTINUE
      DO 520 I=1,3
IS=I+5

```

TABLE A-1 (CONTINUED)

```

VAL(I,1)=.5*X(1,IS)
VAL(I,2)=    X(2,IS)
BK=1.
FK=1.
DO 510 K=3,N2,2
BK=-BK
FK= FK+2.
VAL(I,1)=VAL(I,1)+BK*X(K,IS)
VAL(I,2)=VAL(I,2)+BK*FK*X(K+1,IS)
510 CONTINUE
VAL(I,2)=VAL(I,2)/EKK
520 CONTINUE
WRITE (6,41) ITITLE
WRITE (6,7)
WRITE (6, 8) ((VAL(I,J),I=1,3),J=1,2)
WRITE (6,29)
WRITE (6, 8)((VALZ(I,J),I=1,3),J=1,2)

C
C CONSTANTS OF INTEGRATION FOR INITIAL CONDITIONS
C
E=SGZ
SINC=DSIN(E)
COSC=DCOS(E)
DO 542 I=1,3
ESE=SE*SINC
ECE=1.-SE*COSC
BK=(SGZ-E+ESE)/ECE
E=E+BK-.5*BK*BK*ESE/ECE
SINC=DSIN(E)
COSC=DCOS(E)
542 CONTINUE
CN1=COSC/(1.-SE*COSC)
CN3=SINC/(1.-SE*COSC)
CN2=-SINC
CN4=COSC-SE
DN1=VALZ(3,1)-VAL(3,1)
DN2=VALZ(3,2)-VAL(3,2)
ZK(5)=DN1*CN1+DN2*CN2
ZK(6)=DN1*CN3+DN2*CN4
DN1=VALZ(1,1)-VAL(1,1)
DN2=VALZ(1,2)-VAL(1,2)
ZK(3)=2.*DN1+VALZ(2,2)-VAL(2,2)
DN1=DN1-2.*ZK(3)
ZK(1)=DN1*CN1+DN2*CN2
ZK(2)=DN1*CN3+DN2*CN4
SIN2C=2.*SINC*COSC
COS2C=1.-2.*SINC**2
CNN=-2.+SE*SE
ZK(4)=VALZ(2,1)-VAL(2,1)+(-CNN*SINC-.5*SE*SIN2C)*ZK(1)
1           +(-2.*COSC+.5*SE*COS2C)*ZK(2)
      WRITE (6,13)(I,I=1,6)
      WRITE (6, 8)(ZK(I),I=1,6)

C
C ADD CONTRIBUTIONS FROM CONSTANTS TO SERIES COEFFICIENTS
C
X(1,7)=X(1,7)+2.*ZK(4)
X(2,7)=X(2,7)+3.*((SE*ZK(1)-ZK(3))*EKK
X(1,6)=X(1,6)+4.*ZK(3)-TWOSE*ZK(1)
X(1,8)=X(1,8)           -TWOSE*ZK(5)
X(1,4)=X(1,4)+TWOSE
CALL MLTPLY (4,4,1)

```

TABLE A-1 (CONTINUED)

```

X(1,1)=X(1,1)-1.
C X(K,1) IS .5*COS(2E)
    CALL MLTPLY (4,5,2)
C X(K,2) IS .5*SIN(2E)
    DO 550 K=1,NTIM
        X(K,6)=X(K,6)+ZK(1)*X(K,4)+ZK(2)*X(K,5)
        X(K,8)=X(K,8)+ZK(5)*X(K,4)+ZK(6)*X(K,5)
        X(K,7)=X(K,7)+ZK(1)*(CNN*X(K,5)+SE*X(K,2))
        1           +ZK(2)*( 2.*X(K,4)-SE*X(K,1))
    550 CONTINUE
C
C PRINT AND PUNCH COEFFICIENTS
C
    WRITE (6,43)
    WRITE (6,41) ITITLE
    WRITE (6,9)
    DO 570 K=1,NTIM,2
        K1=K-1
        L=K+1
        DO 564 I=1,3
            J=I+3
            AA(I)=X(K,I+5)
            AA(J)=X(L,I+5)
    564 CONTINUE
        WRITE (6,10) K1,(AA(I),I=1,3),K,(AA(I),I=4,6)
        GO TO NPUNCH ,(568,570)
    568 PUNCH      5,K1,(AA(I),I=1,6)
    570 CONTINUE
C
C EVALUATE SERIES AT EPOCH (SHOULD EQUAL INITIAL CONDITIONS)
C
    DO 580 I=1,3
    DO 580 J=1,2
        VAL(I,J)=0.
    580 CONTINUE
    DO 600 I=1,3
        IS=I+5
        VAL(I,1)=.5*X(1,IS)
        VAL(I,2)=    X(2,IS)
        BK=1.
        FK=1.
        DO 590 K=3,N2,2
            BK=-BK
            FK= FK+2.
            VAL(I,1)=VAL(I,1)+BK*X(K,IS)
            VAL(I,2)=VAL(I,2)+BK*FK*X(K+1,IS)
    590 CONTINUE
        VAL(I,2)=VAL(I,2)/EKK
    600 CONTINUE
        WRITE (6,41) ITITLE
        WRITE (6,7)
        WRITE (6, 8) ((VAL(I,J),I=1,3),J=1,2)
        DO 602 I=1,6
            ZK(I)=0.
    602 CONTINUE
        ITTRM=ITTRM+1
        IF (ITTRM-ITRM) 94,610,604
    604 PUNCH 3
        PUNCH 3
        PUNCH 3
        GO TO 55

```

TABLE A-1 (CONTINUED)

```

610 IF (IPUNCH-1) 94,94,620
620 ASSIGN 568 TO NPUNCH
      GO TO 94
      END

```

```

SUBROUTINE PQRDP2(SU,SI,CO,P,Q,R,EP)
C COMPUTE VECTORS P,Q,R
DOUBLE PRECISION SU,SI,CO,P,Q,R,EP,EPC,EPS,CCO,SCO,CSO,SSO,
X   CSI,SSI,TP1,TP2,DCOS,DSIN
DIMENSION P(3),Q(3),R(3)
C EP IS OBLIQUITY OF ECLIPTIC
EPC=DCOS(EP)
EPS=DSIN(EP)
CCO=DCOS(CO)
SCO=DSIN(CO)
CSO=DCOS(SO)
SSO=DSIN(SO)
CSI=DCOS(SI)
SSI=DSIN(SI)
P(1)=-CSI*SSO*SCO+CSO*CCO
TP1 =+CSI*SSO*CCO+CSO*SCO
TP2 =+SSI*SSO
P(2)=EPC*TP1-EPS*TP2
P(3)=EPS*TP1+EPC*TP2
Q(1)=-CSI*CSO*SCO-SSO*CCO
TP1 = CSI*CSO*CCO-SSO*SCO
TP2 = SSI*CSO
Q(2)=EPC*TP1-EPS*TP2
Q(3)=EPS*TP1+EPC*TP2
R(1)= SSI*SCO
TP1 =-SSI*CCO
TP2 = CSI
R(2)=EPC*TP1-EPS*TP2
R(3)=EPS*TP1+EPC*TP2
RETURN
END

```

TABLE A-1 (CONTINUED)

```

SUBROUTINE FFD (ITYPE)
C SUBROUTINE FOR COMPUTING SPECIAL VALUES OF DISTURBING FORCE
  DOUBLE PRECISION EKK,XJ,V,SA,SB,SE,SN,SGZ,SM,EP,P,Q,R,EK,RAD,X,
  1SAB,P7,P9,P11,P13,SA2,C11,C31,SAP,SEP,SUP2,SIP2,CUP2,SMP2,SGZP2,
  2SMINNK,SUP,CUP,SIP,SGZP,SNP,SNP2,PPP,QPP,RPP,SBP,AP,BP,CP,PP,QP,
  3RP,SMP,SMPA2,EKP,ENT,SG,E,SNE,CSE,DCOS,DSIN,DSQRT,ESE,ECE,BK,COSE,
  4SINE,SR,SR2,SR3,ADR,TADR,SRZ,S1,S2,EX,SRV,F,ENTP,SGP,CK1,CK2,CK3,
  5SPZ,RHU2,SPR2,SPV,RHU,CRHU,CSRP,TRATIO,DZR,XJP,ZK,CS1,CS2,SS2,
  6HALFSE,SEE
    COMMON X,EKK,NTRM,N1,XJ,V,SA,SB,SE,SN,SGZ,SM,EP,P,Q,R,IPRINT,NTIM
    COMMON TRATIO,DZR,ZK
    DIMENSION SEP(9),SGZP(9),PPP(3),QPP(3),RPP(3),AP(3,9),BP(3,9),
    1CP(3,9),PP(3),QP(3),RP(3),SMPA2(9),X( 701,8), Y(3,1200), AA(6),
    2NTPM(9),SRZ(2),SW(2),S1(2),S2(2),PERT(3),S(3),SRV(3),F(3),EKP(9),
    3SPZ(3),SWZ(3),SPV(3),RHU(3),ITITLE(12),P(3),Q(3),R(3),V(5),
    4NFIRST(9),ZK(6)
1 FORMAT (12A6)
2 FORMAT (1H1,12A6,4H M=,I2)
3 FORMAT (4D18.11)
4 FORMAT (12HJSMALL OMEGAF15.7,F13.7,15H ) ECLIPTIC AN05X7HSMALL E
41   F12.8/12H CAP OMEGAF15.7,F13.7,15H ) MEAN EQUINOX5X7HSMALL A
42   F11.7,6X2HAU/8H SMALL IF19.7,F13.7,11H ) 1950.09X7HSMALL N
43   E16.8,8H DEG/DAY/7H G ZEROF20.7,F13.7,20X7HSMALL ME16.8)
5 FORMAT (1HK7X1HA13X1HB13X1HC10X7H P 7X7H Q 7X7H R //)
6 FORMAT (6F14.8,15H ) EQUATOR AND/
61      6F14.8,15H ) MEAN EQUINOX/
62      6F14.8,12H ) 1950.0)
7 FORMAT (1H0)
8 FORMAT (6F14.8,15H ) P,Q,R,SYSTEM/
81      6F14.8, 9H ) UF/
82      6F14.8,15H ) COORDINATES)
9 FORMAT (I8,6P6F12.4)
10 FORMAT (/2(7X1HK1X11HALPHA*1.E6 2X10HBETA*1.E6 1X11HGAMMA*1.E6 )6X
1017HSTORAGE//)
11 FORMAT (58H0 K A**2*F(I) SUM ALPHA,BETa,GAMMA E,COS(E),SIN
111(E)6X6HSRV(I)12X6HSRZ(I)13X5HSW(I))
12 FORMAT (1H0,13,1PD18.10,E18.7,5D18.10)
13 FORMAT (4X,1PD18.10,E18.7,5D18.10)
14 FORMAT (25H0 M DISTURBING PLANETS94X6HRHU(I))
16 FORMAT (2(18,6P3F12.4),218)
17 FORMAT (1H017X7HDEGREES7X7HRADIANS)
     IF (ITYPE-2)100,300,300
C
C COMPUTE CONSTANTS AND READ DISTURBING PLANET DATA
C
100 CONTINUE
  KEPITR=3
  IF (SE-0.25)104,102,102
102 KEPITR=4
104 CONTINUE
  RAD=2.*3.141592653589793/360.
  SAB=SA/SB
  EK=0.01720209895D+0
  P 7= 7./ 6.
  P 9= 9./ 8.
  P11=11./10.
  P13=13./12.
  SA2=SA*SA
  HALFSE=SE*.5
  SEE=-2.+SE**2
  C11=1.5*SA2

```

TABLE A-1 (CONTINUED)

```

C31=-15.*SA2/8.
M=0
K2=0
C ENTRY FOR SUCCESSIVE DISTURBING PLANETS
C
110 CONTINUE
M=M+1
READ (5,1) ITITLE
WRITE(6,2) ITITLE,M
READ (5,3) SAP,SEP(M),SOP2,SIP2,COP2,SMP2,SGZP2,SMINNR
C ELEMENTS OF DISTURBING PLANETS IN SAME FORM AS DISTURBED PLANET
C
SOP=SOP2    *RAD
COP=COP2    *RAD
SIP=SIP2    *RAD
SGZP(M)=SGZP2    *RAD
SNP=EK*DSQRT(1.+SMINNR+SMP2)/SAP**1.5
SNP2 =SNP    /RAD
WRITE (6,17)
WRITE(6,4) SOP2,SOP,SEP(M),COP2,COP,SAP,SIP2,SIP,SNP2,SGZP2,
1SGZP(M),SMP2
C CALL PWRDP2 (SOP,SIP,COP,PPP,QPP,RPP,EP)          PQRDP2
SBP=SAP    *DSQRT (1.-SEP(M)**2)
DO 120 I=1,3
AP(I,M)=SAP    *PPP(I)
BP(I,M)=SBP    *QPP(I)
CP(I,M)=SAP    *RPP(I)
120 CONTINUE
WRITE(6,5)
WRITE(6,6)(AP(I,M),BP(I,M),CP(I,M),PPP(I),QPP(I),RPP(I),I=1,3)
DO 130 I=1,3
PP(I )=0.
QP(I )=0.
RP(I )=0.
130 CONTINUE
DO 140 J=1,3
PP(1 )=PP(1 )+P(J)*PPP(J)
QP(1 )=QP(1 )+P(J)*QPP(J)
RP(1 )=RP(1 )+P(J)*RPP(J)
PP(2 )=PP(2 )+Q(J)*PPP(J)
QP(2 )=QP(2 )+Q(J)*QPP(J)
RP(2 )=RP(2 )+Q(J)*RPP(J)
PP(3 )=PP(3 )+R(J)*PPP(J)
QP(3 )=QP(3 )+R(J)*QPP(J)
RP(3 )=RP(3 )+R(J)*RPP(J)
140 CONTINUE
DO 150 I=1,3
AP(I,M)=SAP    *PP(I)
BP(I,M)=SBP    *QP(I)
CP(I,M)=SAP    *RP(I)
150 CONTINUE
WRITE(6,7)
WRITE(6,8)(AP(I,M),BP(I,M),CP(I,M),PP(I ),QP(I ),RP(I ),I=1,3)
SMP=SMP2/(1.+SMINNR+SM)
SMPA2(M)=SA2*SMP
EKP(M)=EKK*SNP    /(SN*TRATIO)
C READ PERTURBATIONS. ALL COEFFICIENTS STORED IN Y ARRAY

```

TABLE A-1 (CONTINUED)

```

C
      NFIRST(M)=K2+1
      NTPM(M)=0
      WRITE(6,10)
190 READ (5,9) K,(AA(I),I=1,6)
      KK=K+1
      KP=K+NFIRST(M)
      KP1=KP+1
      WRITE(6,16)K,(AA(I),I=1,3),KK,(AA(I),I=4,6),KP,KP1
      IF (K-100)200,110,220
200 K1=KP
      K2=KP1
      IF (K2-1200)202,202,190
202 CONTINUE
      NTPM(M)=K+2
      DO 210 I=1,3
      J=I+3
      Y(I,K1) =AA(I)
      Y(I,K2) =AA(J)
210 CONTINUE
      GO TO 190
220 NPLNET=M
      K=0
      RETURN

C
C COMPUTE SPECIAL VALUES
C
300 CONTINUE
      K=K+1

C
C COMPUTE POSITION VECTOR OF DISTURBED PLANET
C
      ENT=EKK*XJ
      SG=SGZ+ENT
      E=SG
      SNE=DSIN(SG )
      CSE=DCOS(SG )
      DO 310 I=1,KEPITR
      ESE=SNE*SE
      ECE=1.-CSE*SE
      BK=(SG -E+ESE)/ECE
      E=E+BK-.5*BK*BK*ESE/ECE
      CSÉ=DCOS(E)
      SNE=DSIN(E)
310 CONTINUE
      COSE=CSE
      SINE=SNE
      CS1=COSE-SE
      CS2=2.*COSE-SE*(.5-SINE**2)
      SS2=SEE*SINE+SE*(SINE*COSE+3.*ENT)
      V(4)=COSE
      V(5)=SINE
      SR =SA*(1.-SE*COSE)
      SR2=SR*SR
      SR3=SR*SR2
      ADR=SA/SR
      TADR=2.*ADR
      SRZ(1)=SA*CS1
      SRZ(2)=SB*SINE
      SW(1)=-SA*ADR*SINE
      SW(2)= SB*ADR*COSE

```

TABLE A-1 (CONTINUED)

```

S1(1)= ADR*COS&
S1(2)= ADR*SINE*SAB
S2(1)= -TADR*SINE
S2(2)= TADR*CS1*SAB
ALPHA=2.*ZK(3)+ZK(1)*CS1+ZK(2)*SINE
GAMMA= ZK(5)*CS1+ZK(6)*SINE
BETA =ZK(4)+ZK(1)*SS2+ZK(2)*CS2-3.*ENT*ZK(3)
C
C NTIM IS THE NUMBER OF TERMS IN THE SERIES FOR THE PERTURBATIONS
C
EX=2.*XJ
IF (NTIM-1)370,330,340
330 ALPHA=.5*X(1,6) +ALPHA
BETA =.5*X(1,7) +BETA
GAMMA=.5*X(1,8) +GAMMA
GO TO 370
340 CONTINUE
DO 360 IS=6,8
B2=0.
B1=0.
NTR=NTIM-1
DO 350 I=1,NTR
I1=NTR-I+2
BZ=EX*B1-B2+X(I1,IS)
B2=B1
B1=BZ
350 CONTINUE
JS=IS-5
PERT(JS)=.5*(EX*B1-2.*B2+X(1,IS))
360 CONTINUE
ALPHA=PERT(1) +ALPHA
BETA =PERT(2) +BETA
GAMMA=PERT(3) +GAMMA
370 CONTINUE
S(1)=ALPHA*SRZ(1)+BETA*SW(1)
S(2)=ALPHA*SRZ(2)+BETA*SW(2)
S(3)=GAMMA*SA
C
C EXPAND SOLAR TERMS IN POWERS OF PERTURBATIONS
C
SQ=S(1)*S(1)+S(2)*S(2)+S(3)*S(3)
SRV(1)=SRZ(1)+S(1)
SRV(2)=SRZ(2)+S(2)
SRV(3)= S(3)
DEL=(2.*((SRZ(1)*S(1)+SRZ(2)*S(2))+SQ)/SR2
C1=C11*SQ/(SR2*SR3)
C2=C11*DEL/SR3
C3=C31*DEL**2*(1.-P7*DEL*(1.-P9*DEL*(1.-P11*DEL*(1.-P13*DEL))))/SR
13
F(1)=C1*SRZ(1)+C2*S(1)+C3*SRV(1)
F(2)=C1*SRZ(2)+C2*S(2)+C3*SRV(2)
F(3)= C2*S(3)+C3*SRV(3)
IF (IPRINT-3) 372,371,371
371 WRITE (6,11)
WRITE (6,12) K,F(1),ALPHA, E,SRV(1),SRZ(1),SW(1)
WRITE (6,13) F(2), BETA,COS&,SRV(2),SRZ(2),SW(2)
WRITE (6,13) F(3),GAMMA,SINE,SRV(3),SRZ(3),SW(3)
372 CONTINUE
C
C NEXT TAKE THE DISTURBING PLANETS
C

```

TABLE A-1 (CONTINUED)

```

IF (IPRINT-3) 376,374,374
374 WRITE (6,14)
376 XJP=XJ*TRATIO+DXZR
EX=2.*XJP
DO 480 M=1,NPLNET
ENTP=EKP(M)*XJP
SGP=SGZP(M)+ENTP
E=SGP
SNE=DSIN(SGP)
CSE=DCOS(SGP)
DO 380 I=1,3
ESE=SNE*SEP(M)
ECE=1.-CSE*SEP(M)
BK=(SGP-E+ESE)/ECE
E=E+BK-.5*BK*BK*ESE/ECE
CSE=DCOS(E)
SNE=DSIN(E)
380 CONTINUE
CK1=CSE-SEP(M)
CK2=-SNE/(1.-SEP(M)*CSE)
CK3=CSE/(1.-SEP(M)*CSE)
DO 390 I=1,3
SPZ(I)=AP(I,M)*CK1+BP(I,M)*SNE
SWZ(I)=AP(I,M)*CK2+BP(I,M)*CK3
390 CONTINUE
IF (NTPM(M)-1) 400,410,420
400 ALP=0.
BEP=0.
GAP=0.
GO TO 450
410 JPP=NFIRST(M)
ALP=.5*Y(1,JPP)
BEP=.5*Y(2,JPP)
GAP=.5*Y(3,JPP)
GO TO 450
420 CONTINUE
NTP=NTPM(M)-1
DO 440 IS=1,3
B2=0.
B1=0.
DO 430 I=1,NTP
I1=NTP-I+2
JPP=NFIRST(M)+I1-1
BZ=EX*B1-B2+Y(IS,JPP)
B2=B1
B1=BZ
430 CONTINUE
JPP=NFIRST(M)
PERT(IS)=.5*(EX*B1-2.*B2+Y(IS,JPP))
440 CONTINUE
ALP=PERT(1)
BEP=PERT(2)
GAP=PERT(3)
450 CONTINUE
RHO2=0.
SRP2=0.
DO 460 I=1,3
SPV(I)=(1.+ALP)*SPZ(I)+BEP*SWZ(I)+GAP*CP(I,M)
RHO(I)=SPV(I)-SRV(I)
RHO2=RHO2+RHO(I)*RHO(I)
SRP2=SRP2+SPV(I)*SPV(I)

```

TABLE A-1 (CONTINUED)

```

460 CONTINUE
CRHO=SMPA2(M)/(RHO2*DSQRT(RHO2))
CSRP=SMPA2(M)/(SRP2*DSQRT(SRP2))
DO 470 I=1,3
F(I)=F(I)+CRHO*RHO(I)-CSRP*SPV(I)
470 CONTINUE
IF (IPRINT-3)480,471,471
471 WRITE (6,12) M,F(1),ALP, E,SPV(1),SPZ(1),SWZ(1),RHO(1)
WRITE (6,13) F(2),BEP,CSE,SPV(2),SPZ(2),SWZ(2),RHO(2)
WRITE (6,13) F(3),GAP,SNE,SPV(3),SPZ(3),SWZ(3),RHO(3)
480 CONTINUE
V(1) =F(1)*S1(1)+F(2)*S1(2)
V(2) =F(1)*S2(1)+F(2)*S2(2)
V(3) =F(3)
RETURN
END

```

```

SUBROUTINE COEFF
C COMPUTE CONTRIBUTIONS OF SPECIAL VALUES TO SERIES COEFFICIENTS
DOUBLE PRECISION XJ,V,X, TZ,T1,T2,TWOX,EKK
DIMENSION V(5),X( 701,8)
COMMON X,EKK,NTRM,N1,XJ,V
TZ=1.
T1=XJ
DO 100 I=1,5
X (1,I)=X (1,I)+V(I)
X (2,I)=X (2,I)+V(I)*T1
100 CONTINUE
TWOX=2.*XJ
DO 120 K=3,NTRM
T2=TWOX*T1-TZ
DO 110 I=1,5
X (K,I)=X (K,I)+V(I)*T2
110 CONTINUE
TZ=T1
T1=T2
120 CONTINUE
RETURN
END

```

TABLE A-1 (CONTINUED)

```

SUBROUTINE MLTPLY (I1,I2,I3)
C  PERFORM CHEBYSHEV SERIES MULTIPLICATION
C  THE FACTORS ARE IN X(K,J1) AND X(K,J2). THE PRODUCT GOES IN X(K,J3).
COMMON X( 701,8),EKK,N
DOUBLE PRECISION EKK,X,EX
J1=I1
J2=I2
J3=I3
DO 100 K=1, 701
X(K,J3)=0.
100 CONTINUE
N1=N
DO 130 K=1,N1
EX =0.
IM=N-K+1
DO 120 I=1,IM
L=I+K-1
EX = EX +X(I,J1)*X(L,J2)+X(L,J1)*X(I,J2)
120 CONTINUE
X(K,J3)=.5*EX
130 CONTINUE
X(1,J3)=X(1,J3)-.5*X(1,J1)*X(1,J2)
DO 150 K=3,N1
EX =0.
IM=K-1
DO 140 I=2,IM
L=K-I+1
EX = EX +X(I,J1)*X(L,J2)
140 CONTINUE
X(K,J3)=.5*EX +X(K,J3)
150 CONTINUE
RETURN
END

```

```

SUBROUTINE NTGRT (J1,J2)
C  PERFORM CHEBYSHEV SERIES INTEGRATION
C  THE INTEGRAND IS STORED IN X(K,J1) AND THE INTEGRAL IN X(K,J2)
C  THE FACTOR EKK COMES FROM D(NT) = EKK * D(X)
COMMON X( 701,8),EKK,N
DOUBLE PRECISION EKK,X,EK2,EK
I=J1
J=J2
X(1,J)=0.
EK2=2./EKK
EK=0.
DO 100 K=2,N
EK=EK + EK2
X(K,J)= (X(K-1,I)-X(K+1,I))/EK
100 CONTINUE
X(N+1,J)= X(N,I)/(EK+EK2)
RETURN
END

```



Appendix B

Motions of the Five Outer Planets

The coefficients in the Chebyshev Series for the perturbations of the five outer planets computed by the CBY program are tabulated in units of the sixth decimal of a radian. They may be converted to seconds of arc by applying the factor 0.20626 48062. The coefficients are to be multiplied by the Chebyshev polynomials $T_k(x)$ and summed over k taking care to include the factor 1/2 for the coefficients of $T_0(x)$. Letting JD be the Julian Date for which the series are to be evaluated, the independent variable x is defined to be

$$x = \frac{JD - 2415200.5}{36800}.$$

The series are most readily evaluated using Clenshaw's procedure (page 4).

The reference elements for the planets are given in Table 1 on page 17. The position vector is computed using the formula

$$\vec{r} = (1 + \alpha) \vec{r}_0 + \beta \vec{w} + \gamma a \vec{R}$$

where

$$\vec{r}_0 = a \vec{P} (\cos \epsilon - e) + a \sqrt{1 - e^2} \vec{Q} \sin \epsilon$$

$$\vec{w} = -a \vec{P} \frac{\sin \epsilon}{1 - e \cos \epsilon} + a \sqrt{1 - e^2} \vec{Q} \frac{\cos \epsilon}{1 - e \cos \epsilon}.$$

The vectors, \vec{P} , \vec{Q} and \vec{R} are the usual unit vectors of the reference ellipse, and a , e , and ϵ are the semi-major axis, the eccentricity, and the eccentric anomaly respectively from the reference elements. The quantities α , β and γ are the values of the perturbation components obtained from the series. The FFD subroutine described in Appendix A contains a routine for evaluating the position vector \vec{r} (designated as SRV(I) on page 38).



Table B-1

Perturbations of Jupiter.

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
0	41.0068	-964.6742	10.3494	1	-50.3624	-127.7253	-11.9075
2	60.0357	-817.3009	9.6772	3	-92.9013	-43.4000	-11.4463
4	56.1745	-254.3264	8.3875	5	-117.8735	-213.1945	-13.5004
6	30.4887	-6.5151	6.0424	7	-113.3669	-196.1025	-14.0284
8	-1.7558	227.6389	2.8009	9	-54.3652	46.9609	-11.6129
10	10.9379	-205.5816	-1.7433	11	-112.2293	-240.7871	-13.8608
12	-48.6814	83.3622	-6.9742	13	-84.6521	-45.8933	-10.1755
14	-62.4461	36.7154	-11.3516	15	-78.5943	58.8386	-5.9003
16	-49.1141	129.0287	-13.5748	17	-42.9846	203.7106	2.6959
18	16.7849	203.1693	-10.3020	19	23.2458	234.2475	12.1237
20	106.8185	150.8379	-0.7374	21	76.2245	-9.8271	11.6506
22	102.5929	55.8459	7.8830	23	95.7224	-200.2563	6.6656
24	15.3416	-124.6497	12.3684	25	36.0233	-122.5103	-5.4107
26	-37.5174	-119.7082	6.5005	27	-20.3804	-45.2421	-11.4359
28	-54.1821	-19.5208	-5.5719	29	-23.7197	-86.6018	-5.4267
30	-105.8205	-31.5345	-12.2502	31	-41.2314	253.1597	5.9139
32	41.8078	86.2014	-3.2118	33	12.0688	14.5463	10.0773
34	38.0307	-9.2830	9.8041	35	2.1234	87.5261	0.0915
36	101.9230	25.2031	7.9056	37	18.8944	-175.1915	-10.3064
38	-10.0253	11.1649	-8.0801	39	30.7930	-259.7033	-2.5678
40	-206.2312	-105.0949	-8.0610	41	-43.5636	379.8159	11.5993
42	85.6049	-19.4592	10.1534	43	-63.3686	68.1574	-4.3748
44	131.4694	161.9853	1.5852	45	63.3958	-235.6212	-5.9795
46	-56.5534	19.2558	-8.8873	47	82.2052	-71.1815	8.7577
48	-113.9580	-231.7505	6.9647	49	-118.1470	245.5480	-3.5167
50	101.9795	132.6225	-1.2767	51	3.2582	-91.4539	-2.3500
52	12.3426	81.4299	-3.1731	53	74.5184	-76.4076	4.2822
54	-54.6286	-128.8878	3.7752	55	-48.8866	57.6334	-3.5656
56	-0.8246	30.2461	-2.1172	57	-18.3180	82.8196	1.6722
58	85.8566	101.3808	1.1932	59	87.6661	-217.4124	-0.1243
60	-142.5251	-196.3644	-0.9016	61	-119.6170	264.7143	0.0001
62	140.6076	195.3193	0.3045	63	91.2678	-210.3814	-0.0810
64	-92.1606	-120.8940	-0.0573	65	-46.5239	119.7310	-0.2769
66	46.0670	60.4510	0.2609	67	26.1330	-58.1602	0.2122
68	-22.4135	-36.2147	-0.1282	69	-15.0708	20.8873	-0.0047
70	2.5940	11.3696	-0.2539	71	-2.6360	11.7641	0.3271
72	15.1087	13.6367	0.4072	73	11.4880	-25.1688	-0.5801
74	-14.0073	-14.2821	-0.4180	75	-4.8539	12.6865	0.2467
76	1.4038	0.0326	0.3743	77	-3.4586	1.7202	0.1181
78	3.1821	8.1046	-0.0560	79	6.0530	-3.7889	-0.0579
80	-0.7673	-6.5837	-0.3939	81	-2.2715	1.0458	-0.0593
82	0.0172	-4.0928	0.4837	83	-8.0865	1.1958	-0.1304
84	0.5087	17.0122	-0.2533	85	13.8119	-3.2505	0.3655
86	-2.1887	-13.7247	0.1356	87	-3.4056	1.8081	-0.2664
88	0.2199	-6.5770	-0.1433	89	-11.8829	5.2707	-0.0735
90	6.1467	20.6774	0.0854	91	15.4827	-13.0274	0.2848
92	-11.1558	-19.3067	-0.0377	93	-11.5334	15.5736	-0.2097
94	10.8666	15.1854	0.0437	95	10.0323	-12.1095	-0.0115
96	-6.3916	-13.7407	0.0010	97	-8.8053	5.6693	0.1593
98	1.0327	9.4454	-0.0497	99	4.4375	0.0581	-0.1407
100	2.2057	-2.7382	0.0309	101	-0.0066	-2.5806	0.0330
102	-2.0042	-1.9774	-0.0418	103	-2.5208	1.7663	0.0514
104	-0.0494	3.8788	0.1039	105	2.9298	-0.0565	-0.0916
106	0.8534	-2.2503	-0.0931	107	-0.1186	-0.0081	0.1054
108	0.5323	-2.2460	0.0042	109	-3.3024	-1.4600	-0.0803
110	-2.3731	4.1177	0.0560	111	2.7004	2.7967	0.0280
112	3.4051	-0.9590	-0.0774	113	0.4281	-3.4803	0.0033

TABLE B-1 (CONTINUED)

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
114	-3.8685	-2.0228	0.0990	115	-1.2100	3.7220	-0.0027
116	3.6750	1.1441	-0.0937	117	-0.3980	-3.0528	-0.0046
118	-2.4109	1.1631	0.0476	119	1.7274	1.3798	0.0084
120	0.6711	-2.7535	-0.0061	121	-2.6468	0.3242	-0.0052
122	0.6635	4.0900	-0.0082	123	3.6265	-1.3755	-0.0089
124	-1.3730	-4.8812	0.0088	125	-3.7969	1.6904	0.0160
126	1.2817	4.4583	0.0005	127	3.0393	-1.1252	-0.0023
128	-0.4162	-3.3282	-0.0100	129	-1.9887	0.1276	-0.0114
130	-0.2551	1.7352	-0.0014	131	0.5624	0.2150	0.0093
132	0.0355	0.2884	0.0249	133	0.8631	0.1367	-0.0068
134	0.3423	-1.4341	-0.0342	135	-1.0503	-0.1887	0.0115
136	-0.1529	0.7835	0.0297	137	0.1027	-0.1931	-0.0116
138	-0.1835	0.3688	-0.0229	139	0.4995	0.3455	0.0071
140	0.2629	-0.5500	0.0091	141	-0.2530	-0.2560	-0.0057
142	-0.3337	0.0129	0.0126	143	-0.1820	0.2929	0.0042
144	0.4715	0.4591	-0.0281	145	0.4404	-0.3788	-0.0022
146	-0.5058	-0.6300	0.0312	147	-0.4813	0.4040	0.0069
148	0.4928	0.3937	-0.0270	149	0.1495	-0.4197	-0.0117
150	-0.4003	0.2255	0.0172	151	0.4385	0.2578	0.0056
152	0.0809	-0.8682	-0.0046	153	-0.9423	0.0717	0.0032
154	0.1667	1.3272	-0.0027	155	1.2859	-0.1153	-0.0033
156	0.0095	-1.6225	0.0023	157	-1.4177	-0.1877	-0.0007
158	-0.2786	1.5311	-0.0005	159	1.0772	0.3036	0.0017
160	0.1962	-0.8649	0.0014	161	-0.3293	-0.0731	-0.0019
162	0.0531	0.0247	-0.0049	163	-0.2870	-0.1160	0.0016
164	-0.1374	0.4142	0.0105	165	0.4364	0.0923	0.0012
166	0.0855	-0.3752	-0.0156	167	-0.2837	0.0055	-0.0023
168	0.0029	0.1531	0.0150	169	0.0969	-0.1025	0.
170	-0.0967	0.0089	-0.0077	171	-0.0064	0.1358	0.0014
172	0.1016	-0.0105	-0.0019	173	0.0556	-0.0536	-0.0019
174	-0.0420	-0.1180	0.0089	175	-0.1780	0.0065	0.0051
176	0.0858	0.2286	-0.0118	177	0.2303	-0.1036	-0.0082
178	-0.1857	-0.2114	0.0106	179	-0.1592	0.1635	0.0068
180	0.1470	0.0869	-0.0066	181	0.0100	-0.0768	-0.0028
182	-0.0381	0.1047	0.0023	183	0.1933	0.0338	-0.
184	0.0648	-0.3450	0.0002	185	-0.4121	-0.1450	0.0014
186	-0.1892	0.5422	-0.0009	187	0.5241	0.2580	-0.0013
188	0.2411	-0.5626	0.0006	189	-0.4424	-0.2469	-0.0004
190	-0.1827	0.3908	-0.0001	191	0.2357	0.1394	0.0013
192	0.0639	-0.1513	0.0003	193	-0.0387	-0.0032	0.0001
194	0.0514	-0.0295	-0.0022	195	-0.0769	-0.0744	-0.0015
196	-0.0775	0.1050	0.0048	197	0.1015	0.0377	0.0016
198	0.0158	-0.0881	-0.0059	199	-0.0578	0.0305	-0.0016
200	0.0210	0.0301	0.0045	201	0.0065	-0.0266	0.0013
202	0.	0.0016	-0.0016	203	-0.0047	-0.0040	0.0005
204	0.0045	0.0206	-0.0013	205	0.0474	-0.0165	-0.0028
206	-0.0491	-0.0664	0.0033	207	-0.0901	0.0471	0.0042
208	0.0604	0.0952	-0.0035	209	0.1020	-0.0321	-0.0042
210	-0.0293	-0.0851	0.0025	211	-0.0668	0.0071	0.0031
212	0.0150	0.0245	-0.0015	213	-0.0170	-0.0231	-0.0010
214	-0.0444	0.0717	0.0007	215	0.1109	0.0795	-0.0006
216	0.1014	-0.1520	0.	217	-0.1610	-0.1450	0.0007
218	-0.1490	0.1760	-0.0002	219	0.1557	0.1692	-0.0001
220	0.1389	-0.1501	-0.0001	221	-0.1171	-0.1184	-0.0001
222	-0.0675	0.0965	0.0002	223	0.0598	0.0333	0.
224	-0.0014	-0.0303	0.0005	225	-0.0002	0.0143	0.0003
226	0.0223	-0.0227	-0.0015	227	-0.0321	-0.0164	-0.0010
228	-0.0189	0.0374	0.0021	229	0.0274	0.0124	0.0015
230	0.0187	-0.0224	-0.0020	231	-0.0096	-0.0122	-0.0013

TABLE B-1 (CONTINUED)

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
232	-0.0128	0.0064	0.0012	233	0.0025	0.0027	0.0005
234	-0.0043	-0.0055	-0.0001	235	-0.0126	0.0107	0.0007
236	0.0167	0.0204	-0.0006	237	0.0353	-0.0139	-0.0018
238	-0.0148	-0.0412	0.0007	239	-0.0525	0.0049	0.0023
240	0.0035	0.0465	-0.0007	241	0.0432	0.0069	-0.0020
242	0.0043	-0.0255	0.0006	243	-0.0125	-0.0040	0.0010
244	0.0090	-0.0053	-0.0003	245	-0.0157	-0.0267	-0.0002
246	-0.0455	0.0270	0.	247	0.0326	0.0692	-0.0002
248	0.0780	-0.0403	-0.	249	-0.0430	-0.0887	0.0003
250	-0.0794	0.0469	0.0001	251	0.0435	0.0736	-0.0001
252	0.0550	-0.0398	-0.0001	253	-0.0292	-0.0427	-0.0001
254	-0.0260	0.0200	-0.0001	255	0.0087	0.0151	-0.0001
256	0.0030	-0.0001	0.0004	257	0.0058	0.0039	0.0005
258	0.0119	-0.0105	-0.0007	259	-0.0097	-0.0132	-0.0008
260	-0.0163	0.0099	0.0008	261	0.0044	0.0127	0.0009
262	0.0113	-0.0028	-0.0005	263	0.0014	-0.0059	-0.0006
264	-0.0025	0.0004	0.0002	265	0.0035	-0.0019	0.0001
266	-0.0043	-0.0091	-0.0001	267	-0.0165	0.0050	0.0006
268	0.0043	0.0193	0.	269	0.0235	-0.0001	-0.0010
270	0.0029	-0.0196	0.	271	-0.0192	-0.0084	0.0010
272	-0.0086	0.0126	-0.	273	0.0112	0.0087	-0.0008
274	0.0026	-0.0062	0.	275	-0.0049	0.0047	0.0004
276	0.0141	0.0013	-0.	277	-0.0010	-0.0239	-0.
278	-0.0303	0.0039	0.	279	0.0064	0.0371	-0.0001
280	0.0377	-0.0082	-0.0001	281	-0.0093	-0.0390	0.0001
282	-0.0346	0.0095	0.0001	283	0.0085	0.0309	0.
284	0.0235	-0.0068	-0.0001	285	-0.0040	-0.0171	0.
286	-0.0090	0.0009	-0.0001	287	-0.0015	0.0033	-0.0002
288	-0.0027	0.0034	0.0002	289	0.0026	0.0054	0.0004
290	0.0081	-0.0020	-0.0002	291	0.0011	-0.0079	-0.0005
292	-0.0076	-0.0015	0.0001	293	-0.0029	0.0054	0.0004
294	0.0037	0.0006	-0.0001	295	-0.0011	-0.0012	-0.0002
296	0.0001	0.0043	0.0001	297	0.0067	-0.0009	-0.0001
298	-0.0002	-0.0078	-0.0001	299	-0.0091	-0.0012	0.0003
300	-0.0032	0.0077	0.0001	301	0.0083	0.0051	-0.0005
302	0.0059	-0.0059	-0.0001	303	-0.0063	-0.0060	0.0004
304	-0.0045	0.0042	0.0001	305	0.0047	0.0022	-0.0003
306	-0.0011	-0.0036	-0.0001	307	-0.0038	0.0050	0.0001
308	0.0086	0.0034	0.	309	0.0028	-0.0127	0.
310	-0.0151	-0.0021	0.0001	311	-0.0010	0.0174	-0.
312	0.0175	0.0006	-0.0001	313	0.0003	-0.0169	-0.
314	-0.0144	-0.0007	0.0001	315	-0.0013	0.0117	0.
316	0.0079	0.0020	0.	317	0.0020	-0.0048	0.0001
318	-0.0014	-0.0017	-0.	319	-0.0005	-0.0007	-0.0001
320	-0.0027	-0.0004	0.	321	-0.0019	0.0034	0.0002
322	0.0037	0.0019	-0.	323	0.0024	-0.0031	-0.0002
324	-0.0025	-0.0012	-0.	325	-0.0006	0.0016	0.0002
326	0.0011	-0.0010	-0.	327	-0.0019	-0.0007	-0.0001
328	-0.0010	0.0029	0.	329	0.0035	0.0013	-0.0001
330	0.0020	-0.0032	-0.0001	331	-0.0034	-0.0025	0.0002
332	-0.0031	0.0024	0.0001	333	0.0026	0.0033	-0.0002
334	0.0035	-0.0018	-0.0001	335	-0.0024	-0.0029	0.0001
336	-0.0022	0.0021	0.0001	337	0.0026	0.0007	-0.0001
338	-0.0010	-0.0026	-0.	339	-0.0026	0.0031	0.
340	0.0048	0.0026	0.	341	0.0024	-0.0064	-0.
342	-0.0072	-0.0026	0.	343	-0.0026	0.0076	0.
344	0.0071	0.0029	-0.	345	0.0029	-0.0063	-0.
346	-0.0049	-0.0028	0.	347	-0.0022	0.0036	-0.
348	0.0020	0.0017	0.	349	0.0007	-0.0008	0.

TABLE B-1 (CONTINUED)

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
350	0.0003	-0.	-0.	351	0.0009	-0.0008	-0.0001
352	-0.0011	-0.0012	-0.	353	-0.0016	0.0012	0.0001
354	0.0011	0.0012	0.	355	0.0011	-0.0010	-0.0001
356	-0.0010	-0.0003	-0.	357	0.0001	0.0009	0.0001
358	0.0009	-0.0008	-0.	359	-0.0010	-0.0009	-0.
360	-0.0011	0.0011	0.	361	0.0012	0.0011	-0.
362	0.0016	-0.0009	-0.0001	363	-0.0010	-0.0017	0.0001
364	-0.0021	0.0007	0.0001	365	0.0009	0.0020	-0.0001
366	0.0020	-0.0007	-0.0001	367	-0.0010	-0.0015	0.
368	-0.0009	0.0010	0.	369	0.0012	-0.	-0.
370	-0.0008	-0.0014	-0.	371	-0.0016	0.0017	0.
372	0.0023	0.0020	-0.	373	0.0022	-0.0028	-0.
374	-0.0029	-0.0024	0.	375	-0.0024	0.0027	0.
376	0.0023	0.0023	-0.	377	0.0020	-0.0018	-0.
378	-0.0013	-0.0016	0.	379	-0.0010	0.0008	-0.
380	0.0004	0.0005	-0.	381	-0.0001	-0.0001	0.
382	0.0001	0.0004	0.	383	0.0008	-0.0002	-0.
384	-0.0003	-0.0008	-0.	385	-0.0009	0.0005	0.0001
386	0.0005	0.0005	0.	387	0.0004	-0.0005	-0.
388	-0.0004	-0.0001	-0.	389	0.	0.0004	0.
390	0.0005	-0.0002	-0.	391	-0.0003	-0.0005	-0.
392	-0.0008	0.0003	0.	393	0.0003	0.0008	-0.
394	0.0011	-0.0002	-0.	395	-0.0002	-0.0011	0.
396	-0.0013	0.0001	0.	397	0.0001	0.0011	-0.
398	0.0010	-0.0001	-0.	399	-0.0003	-0.0006	0.
400	-0.0003	0.0005	0.	401	0.0007	-0.0002	-0.
402	-0.0005	-0.0010	0.	403	-0.0012	0.0007	0.
404	0.0008	0.0014	-0.	405	0.0015	-0.0008	-0.
406	-0.0008	-0.0015	0.	407	-0.0014	0.0007	0.
408	0.0005	0.0012	-0.	409	0.0009	-0.0004	-0.
410	-0.0003	-0.0007	0.	411	-0.0003	0.0002	-0.
412	0.0001	0.0001	-0.	413	-0.0002	-0.	0.
414	0.	0.0003	0.	415	0.0005	-0.0001	-0.
416	-0.0001	-0.0004	-0.	417	-0.0004	0.0002	0.
418	0.0001	0.0003	0.	419	0.0002	-0.0002	-0.
420	-0.0002	-0.0001	-0.	421	-0.	0.0002	0.
422	0.0003	-0.	-0.	423	-0.	-0.0004	-0.
424	-0.0005	-0.	0.	425	-0.	0.0005	0.
426	0.0007	0.0001	-0.	427	0.0001	-0.0006	-0.
428	-0.0006	-0.0001	0.	429	-0.0001	0.0004	0.
430	0.0003	-0.	-0.	431	-0.0002	-0.0002	0.
432	-0.0001	0.0003	0.	433	0.0005	-0.	-0.
434	-0.0001	-0.0006	0.	435	-0.0007	0.0001	0.
436	0.0001	0.0008	-0.	437	0.0008	-0.0001	-0.
438	-0.0001	-0.0007	0.	439	-0.0007	0.0001	0.
440	0.0001	0.0005	-0.	441	0.0004	-0.0001	0.
442	-0.0001	-0.0002	0.	443	-0.0001	0.	-0.
444	0.	-0.0001	-0.	445	-0.0002	-0.	0.
446	-0.	0.0002	0.	447	0.0003	0.	-0.
448	0.	-0.0002	-0.	449	-0.0002	0.	0.
450	0.	0.0001	0.	451	0.0001	-0.0001	-0.
452	-0.0001	-0.	-0.	453	-0.	0.0001	0.
454	0.0002	0.0001	-0.	455	0.0001	-0.0002	-0.
456	-0.0003	-0.0001	0.	457	-0.0001	0.0003	0.
458	0.0003	0.0001	-0.	459	0.0001	-0.0002	-0.
460	-0.0002	-0.0001	0.	461	-0.	0.0002	0.
462	0.0001	-0.	-0.	463	-0.0001	-0.0001	0.
464	-0.0001	0.0002	0.	465	0.0003	0.0001	-0.
466	0.0001	-0.0003	0.	467	-0.0004	-0.0001	0.

TABLE B-1 (CONTINUED)

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
468	-0.0001	0.0004	-0.	469	0.0004	0.0001	-0.
470	0.	-0.0003	0.	471	-0.0003	-0.	0.
472	-0.	0.0002	-0.	473	0.0001	0.	0.
474	-0.	-0.	0.	475	0.	0.	-0.
476	0.	-0.0001	-0.	477	-0.0001	-0.	0.
478	-0.0001	0.0001	0.	479	0.0001	0.	-0.
480	0.0001	-0.0001	-0.	481	-0.0001	-0.	0.
482	-0.	0.	0.	483	0.0001	-0.	-0.
484	-0.0001	-0.0001	-0.	485	-0.0001	0.0001	0.
486	0.0001	0.0001	-0.	487	0.0001	-0.0001	-0.
488	-0.0001	-0.0001	0.	489	-0.0001	0.0001	0.
490	0.0001	0.0001	-0.	491	0.0001	-0.0001	-0.
492	-0.0001	-0.	0.	493	-0.	0.0001	0.
494	0.0001	-0.	-0.	495	-0.0001	-0.0001	0.
496	-0.0001	0.0001	0.	497	0.0001	0.0001	-0.
498	0.0001	-0.0002	-0.	499	-0.0002	-0.0001	0.
500	-0.0001	0.0002	0.	501	0.0002	0.0001	-0.
502	0.0001	-0.0001	-0.	503	-0.0001	-0.	0.
504	-0.	0.0001	-0.	505	0.	0.	-0.
506	-0.	0.	0.	507	0.	0.	0.
508	0.	-0.	-0.	509	-0.	-0.	-0.
510	-0.	0.	0.	511	0.	0.	0.
512	0.	-0.	-0.	513	-0.	-0.	-0.
514	0.	0.	0.	515	0.	-0.	-0.
516	-0.	-0.	0.	517	-0.0001	0.	0.
518	0.	0.0001	-0.	519	0.0001	-0.	-0.
520	-0.	-0.0001	0.	521	-0.0001	0.	0.
522	0.	0.0001	-0.	523	0.	-0.	-0.
524	-0.	-0.	0.	525	-0.	0.	0.
526	0.	-0.	-0.	527	-0.	-0.	0.
528	-0.0001	0.	0.	529	0.0001	0.0001	-0.
530	0.0001	-0.0001	-0.	531	-0.0001	-0.0001	0.
532	-0.0001	0.0001	0.	533	0.0001	0.	-0.

Table B-2
Perturbations of Saturn.

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
0	757.4862	2557.6035	-37.6214	1	-117.5646	571.6094	3.5885
2	-11.2971	2395.4467	-31.9203	3	146.5709	-37.2696	33.7066
4	-9.0744	967.6583	-26.0238	5	364.2359	485.3840	56.8250
6	97.2528	86.2457	-11.7247	7	408.9010	511.6943	68.9338
8	223.2753	-892.9087	9.2092	9	41.5396	-294.3393	45.0427
10	6.3576	426.4257	31.1112	11	317.0505	579.3184	7.2062
12	140.3957	-659.3633	26.9913	13	-107.3764	-324.9522	-55.9981
14	-186.8598	80.1544	-11.2740	15	-129.0829	-375.9183	-28.7434
16	-410.4045	367.7741	-28.0946	17	80.8977	152.8467	47.8188
18	-30.8178	-149.5045	15.9630	19	-5.1778	240.2706	-4.9858
20	394.7012	-210.2148	5.7539	21	-88.2616	-211.3491	-21.0426
22	-19.2782	163.2504	-11.9351	23	135.9595	134.0736	18.1213
24	-242.1772	-88.2683	5.2517	25	-105.9253	-37.9114	-6.9453
26	-13.0841	23.2219	-0.8876	27	-77.9873	11.5731	2.3811
28	295.7681	38.3075	3.7553	29	136.2915	46.0929	0.1210
30	-284.2382	-2.5616	-0.6644	31	-94.4704	-59.1773	-3.0597
32	231.5979	9.8748	-1.5604	33	75.9000	-51.0938	0.9746
34	-162.0822	-26.1567	-1.9357	35	-34.4429	44.0833	2.0553
36	24.6246	-3.0889	1.8724	37	-14.6697	60.1371	-0.3947
38	48.3761	24.5922	1.1826	39	7.0355	-68.9855	-1.8325
40	-29.2013	6.5140	-1.7606	41	19.7551	32.3549	1.4425
42	26.3000	-15.9477	-0.2820	43	6.2526	-41.3253	0.1882
44	-48.7061	-15.7794	1.5766	45	-18.1907	33.4259	-0.7281
46	16.1266	7.3700	-0.4690	47	-11.6482	9.1741	-0.6533
48	29.2244	20.1881	-1.8199	49	16.7389	-12.5975	1.8763
50	-12.2890	-2.3707	2.8350	51	11.2729	-20.1820	-1.5989
52	-23.1361	-29.3539	-2.0184	53	-24.1171	35.6686	0.9456
54	23.7407	24.2311	1.2514	55	12.7304	-23.9561	-0.7208
56	-3.9919	-3.9732	-1.0072	57	2.0572	-1.7880	0.4760
58	-16.6763	-11.7436	0.4265	59	-22.0305	31.4243	-0.2407
60	37.3260	30.6574	0.0894	61	36.7463	-46.3659	0.2173
62	-43.5741	-36.2562	0.0902	63	-26.3699	35.3281	-0.1330
64	28.9982	20.4505	-0.2405	65	8.7824	-17.8349	0.0161
66	-12.6798	-7.5062	-0.0995	67	-7.5743	10.5821	-0.0218
68	4.1002	6.3830	0.2153	69	8.3292	-4.9285	0.0576
70	2.6652	-1.7969	0.0688	71	2.9347	-5.1309	-0.0708
72	-6.9978	-5.2285	-0.2437	73	-9.3860	10.4452	0.1419
74	4.4825	2.9059	0.2768	75	2.9564	-6.2324	-0.1853
76	0.1208	2.5660	-0.2611	77	2.9690	-0.5725	0.0591
78	-0.8255	-2.0872	-0.0004	79	-3.6468	4.0197	0.1094
80	-1.4029	-0.8942	0.3626	81	2.4900	-3.3743	-0.1540
82	4.3205	4.0762	-0.4191	83	1.9548	0.4065	0.1188
84	-6.1140	-8.0351	0.2659	85	-5.9128	1.2972	-0.0980
86	4.5631	6.8963	-0.2062	87	1.3882	0.4170	0.0854
88	-0.0447	1.2737	0.1573	89	6.7922	-3.6853	-0.0491
90	-4.1323	-7.3972	-0.0157	91	-8.8112	5.5740	-0.0115
92	5.8276	7.2140	-0.0581	93	6.9931	-5.5484	0.0727
94	-5.5047	-6.1604	0.0148	95	-6.7534	4.4543	-0.0837
96	4.0818	6.0151	0.0165	97	5.6603	-2.8439	0.0380
98	-2.0513	-3.8348	-0.0219	99	-2.0149	1.0394	0.0088
100	-0.0232	0.5117	0.0229	101	-0.8396	0.4913	-0.0375
102	1.2579	1.4280	0.0472	103	1.7708	-1.0949	0.0645
104	-1.1830	-2.0885	-0.1442	105	-1.7869	0.7006	-0.0669
106	0.5003	1.2993	0.1283	107	0.2621	-0.1419	0.0329
108	-0.1554	0.8993	-0.0198	109	1.6435	0.0016	-0.0118
110	0.1502	-1.6445	-0.0598	111	-0.9022	-0.0442	0.0231
112	-0.1904	-0.3751	0.0971	113	-1.5290	0.1587	-0.0269

TABLE B-2 (CONTINUED)

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
114	0.4344	2.0622	-0.1227	115	2.1072	-0.4429	0.0138
116	-0.6967	-1.3041	0.1131	117	-0.7209	0.4852	-0.0049
118	0.4079	-0.2729	-0.0685	119	-0.6351	-0.0130	-0.0038
120	0.2644	1.2930	0.0277	121	1.7302	-0.5106	0.0116
122	-0.7395	-2.0701	0.0032	123	-2.7487	0.7636	-0.0022
124	0.9587	2.5113	-0.0245	125	2.9920	-0.8422	-0.0069
126	-0.9155	-2.3212	0.0129	127	-2.4404	0.6220	-0.0083
128	0.4321	1.8264	0.0172	129	1.5697	-0.1201	0.0217
130	0.0851	-1.0286	-0.0169	131	-0.2940	-0.1264	-0.0087
132	-0.1090	-0.1552	-0.0151	133	-0.9513	0.0003	-0.0064
134	-0.0138	0.8364	0.0388	135	1.0608	-0.0177	0.0046
136	-0.1581	-0.3774	-0.0390	137	-0.1906	0.2303	0.
138	0.2969	-0.3570	0.0246	139	-0.3457	-0.2247	-0.0021
140	-0.1541	0.4136	0.0003	141	0.1190	0.0506	0.0075
142	0.0452	-0.0070	-0.0272	143	0.2897	-0.0064	-0.0090
144	-0.0840	-0.3258	0.0417	145	-0.5162	0.0412	0.0061
146	0.0960	0.4321	-0.0425	147	0.4874	-0.0702	-0.0106
148	-0.1228	-0.2534	0.0356	149	-0.1293	0.1385	0.0155
150	0.1650	-0.1591	-0.0208	151	-0.3783	-0.1124	-0.0067
152	-0.0406	0.5451	0.0029	153	0.7608	-0.0574	-0.0047
154	-0.1078	-0.8031	0.0058	155	-1.0183	0.0803	0.0026
156	-0.0075	0.9918	-0.0022	157	1.1302	0.1138	0.0050
158	0.1905	-0.9594	-0.0037	159	-0.8709	-0.1839	-0.0068
160	-0.1211	0.5544	0.0043	161	0.2900	0.0310	0.0048
162	-0.0506	-0.0312	0.0004	163	0.1809	0.0763	-0.0008
164	0.0762	-0.2342	-0.0077	165	-0.2840	-0.0370	-0.0047
166	-0.0139	0.1983	0.0135	167	0.1524	-0.0336	0.0055
168	-0.0434	-0.0540	-0.0135	169	-0.0020	0.0883	-0.0008
170	0.0912	-0.0439	0.0067	171	-0.0515	-0.0936	-0.0020
172	-0.0652	0.0311	0.0032	173	-0.0213	0.0138	0.0027
174	-0.0170	0.0656	-0.0116	175	0.1428	0.0385	-0.0075
176	0.0063	-0.1416	0.0160	177	-0.1903	0.0256	0.0129
178	0.0760	0.1293	-0.0153	179	0.1324	-0.0774	-0.0114
180	-0.0669	-0.0508	0.0101	181	-0.0096	0.0301	0.0051
182	0.0063	-0.0692	-0.0036	183	-0.1604	-0.0132	-0.0002
184	-0.0498	0.2260	-0.0006	185	0.3438	0.1007	-0.0019
186	0.1579	-0.3598	0.0016	187	-0.4389	-0.1805	0.0009
188	-0.1938	0.3764	-0.0011	189	0.3737	0.1697	0.0020
190	0.1403	-0.2614	0.0002	191	-0.1999	-0.0952	-0.0027
192	-0.0451	0.0987	-0.	193	0.0303	0.0030	0.
194	-0.0435	0.0238	0.0016	195	0.0670	0.0482	0.0023
196	0.0582	-0.0731	-0.0043	197	-0.0834	-0.0178	-0.0023
198	-0.0008	0.0593	0.0057	199	0.0455	-0.0325	0.0022
200	-0.0306	-0.0185	-0.0043	201	-0.0041	0.0249	-0.0021
202	0.0049	-0.0038	0.0012	203	0.0020	0.0051	-0.0003
204	0.0058	-0.0105	0.0020	205	-0.0342	0.0037	0.0042
206	0.0168	0.0406	-0.0044	207	0.0668	-0.0223	-0.0068
208	-0.0184	-0.0600	0.0048	209	-0.0766	0.0103	0.0071
210	-0.0039	0.0549	-0.0036	211	0.0513	0.0050	-0.0051
212	0.0032	-0.0159	0.0021	213	0.0138	0.0113	0.0017
214	0.0346	-0.0499	-0.0009	215	-0.0916	-0.0558	0.0006
216	-0.0903	0.1068	-0.0003	217	0.1370	0.1054	-0.0006
218	0.1328	-0.1245	0.0009	219	-0.1344	-0.1238	-0.0003
220	-0.1230	0.1064	-0.0004	221	0.1001	0.0861	0.0005
222	0.0599	-0.0683	-0.0001	223	-0.0493	-0.0234	-0.0001
224	-0.	0.0208	-0.0004	225	-0.0023	-0.0099	-0.0003
226	-0.0174	0.0176	0.0013	227	0.0299	0.0091	0.0010
228	0.0131	-0.0281	-0.0020	229	-0.0260	-0.0054	-0.0018
230	-0.0117	0.0168	0.0020	231	0.0106	0.0062	0.0018

TABLE B-2 (CONTINUED)

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
232	0.0074	-0.0049	-0.0011	233	-0.0033	-0.0009	-0.0007
234	0.0033	0.0032	-0.0001	235	0.0087	-0.0071	-0.0011
236	-0.0086	-0.0125	0.0008	237	-0.0243	0.0076	0.0029
238	0.0031	0.0270	-0.0008	239	0.0377	-0.0001	-0.0038
240	0.0072	-0.0313	0.0008	241	-0.0317	-0.0087	0.0031
242	-0.0118	0.0167	-0.0009	243	0.0081	0.0059	-0.0017
244	-0.0036	0.0050	0.0005	245	0.0148	0.0183	0.0004
246	0.0390	-0.0202	0.	247	-0.0293	-0.0515	0.0001
248	-0.0690	0.0297	-0.0002	249	0.0383	0.0666	-0.0002
250	0.0704	-0.0349	0.0001	251	-0.0384	-0.0548	0.0001
252	-0.0484	0.0297	0.	253	0.0254	0.0314	0.0001
254	0.0222	-0.0147	0.0001	255	-0.0072	-0.0110	0.
256	-0.0020	-0.0004	-0.0004	257	-0.0055	-0.0027	-0.0004
258	-0.0102	0.0085	0.0007	259	0.0094	0.0093	0.0009
260	0.0130	-0.0082	-0.0008	261	-0.0055	-0.0088	-0.0011
262	-0.0086	0.0026	0.0004	263	0.0001	0.0038	0.0008
264	0.0017	-0.0002	-0.0001	265	-0.0026	0.0017	0.
266	0.0029	0.0061	0.0001	267	0.0118	-0.0037	-0.0009
268	-0.0015	-0.0135	-0.0002	269	-0.0168	-0.0005	0.0016
270	-0.0058	0.0134	0.0001	271	0.0132	0.0074	-0.0016
272	0.0108	-0.0081	0.	273	-0.0074	-0.0076	0.0012
274	-0.0046	0.0038	0.	275	0.0029	-0.0031	-0.0006
276	-0.0114	-0.0007	0.	277	0.0017	0.0183	0.0001
278	0.0269	-0.0030	0.	279	-0.0060	-0.0286	0.0002
280	-0.0338	0.0062	0.	281	0.0082	0.0301	-0.0001
282	0.0309	-0.0073	-0.0001	283	-0.0075	-0.0237	0.
284	-0.0207	0.0053	0.	285	0.0036	0.0131	0.
286	0.0078	-0.0007	0.0001	287	0.0012	-0.0025	0.0001
288	0.0024	-0.0028	-0.0003	289	-0.0025	-0.0041	-0.0004
290	-0.0071	0.0018	0.0002	291	-0.0003	0.0060	0.0005
292	0.0066	0.0009	-0.0001	293	0.0017	-0.0040	-0.0005
294	-0.0030	-0.0003	0.	295	0.0012	0.0007	0.0003
296	-0.0002	-0.0033	-0.0001	297	-0.0052	0.0009	0.0001
298	0.	0.0057	0.0002	299	0.0066	0.0010	-0.0005
300	0.0034	-0.0055	-0.0003	301	-0.0056	-0.0041	0.0007
302	-0.0061	0.0039	0.0002	303	0.0040	0.0049	-0.0007
304	0.0048	-0.0027	-0.0002	305	-0.0029	-0.0019	0.0004
306	0.0005	0.0025	0.0001	307	0.0027	-0.0039	-0.0002
308	-0.0077	-0.0026	-0.0001	309	-0.0022	0.0100	0.
310	0.0137	0.0017	0.	311	0.0010	-0.0138	0.
312	-0.0158	-0.0005	0.0001	313	-0.0003	0.0134	0.
314	0.0130	0.0006	0.	315	0.0011	-0.0093	0.
316	-0.0072	-0.0016	0.	317	-0.0016	0.0038	0.
318	0.0013	0.0014	0.0001	319	0.0004	0.0006	0.0001
320	0.0024	0.0002	-0.0001	321	0.0015	-0.0027	-0.0002
322	-0.0034	-0.0014	0.	323	-0.0018	0.0025	0.0003
324	0.0022	0.0008	0.	325	0.0003	-0.0012	-0.0002
326	-0.0009	0.0009	0.	327	0.0016	0.0005	0.0001
328	0.0008	-0.0023	-0.0001	329	-0.0026	-0.0010	0.0001
330	-0.0018	0.0024	0.0002	331	0.0023	0.0019	-0.0002
332	0.0029	-0.0016	-0.0002	333	-0.0015	-0.0026	0.0003
334	-0.0032	0.0011	0.0002	335	0.0014	0.0024	-0.0002
336	0.0020	-0.0015	-0.0002	337	-0.0018	-0.0005	0.0001
338	0.0009	0.0020	0.0001	339	0.0021	-0.0025	0.
340	-0.0044	-0.0020	0.	341	-0.0022	0.0052	0.
342	0.0066	0.0021	0.	343	0.0024	-0.0061	0.
344	-0.0065	-0.0024	0.	345	-0.0026	0.0051	0.
346	0.0045	0.0023	0.	347	0.0020	-0.0029	0.
348	-0.0018	-0.0014	0.	349	-0.0006	0.0006	-0.0001

TABLE B-2 (CONTINUED)

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
350	-0.0002	0.	0.	351	-0.0008	0.0007	0.0001
352	0.0011	0.0009	0.	353	0.0014	-0.0010	-0.0001
354	-0.0011	-0.0009	-0.	355	-0.0009	0.0008	0.0001
356	0.0009	0.0002	0.	357	-0.0002	-0.0007	-0.0001
358	-0.0007	0.0006	0.	359	0.0008	0.0007	0.
360	0.0009	-0.0009	-0.0001	361	-0.0008	-0.0009	0.0001
362	-0.0013	0.0006	0.0001	363	0.0006	0.0014	-0.0001
364	0.0018	-0.0004	-0.0001	365	-0.0005	-0.0016	0.0001
366	-0.0018	0.0005	0.0001	367	0.0006	0.0012	-0.0001
368	0.0007	-0.0007	-0.0001	369	-0.0009	0.	0.
370	0.0008	0.0011	0.	371	0.0014	-0.0014	-0.
372	-0.0022	-0.0016	0.	373	-0.0020	0.0023	0.
374	0.0026	0.0020	-0.	375	0.0022	-0.0022	-0.
376	-0.0021	-0.0019	0.	377	-0.0018	0.0015	-0.
378	0.0012	0.0013	0.	379	0.0009	-0.0007	0.
380	-0.0004	-0.0004	-0.	381	0.0001	0.0001	-0.
382	-0.0001	-0.0003	-0.	383	-0.0007	0.0002	0.
384	0.0003	0.0006	0.	385	0.0007	-0.0004	-0.0001
386	-0.0004	-0.0004	-0.	387	-0.0003	0.0004	0.0001
388	0.0004	0.	0.	389	-0.	-0.0004	-0.
390	-0.0004	0.0002	0.	391	0.0002	0.0004	0.
392	0.0006	-0.0002	-0.	393	-0.0002	-0.0007	0.
394	-0.0009	0.0001	0.0001	395	0.	0.0009	-0.
396	0.0011	-0.	-0.0001	397	0.	-0.0009	0.
398	-0.0008	0.0001	0.0001	399	0.0002	0.0005	-0.
400	0.0002	-0.0004	-0.	401	-0.0006	0.0002	0.
402	0.0005	0.0008	0.	403	0.0011	-0.0006	-0.
404	-0.0008	-0.0011	0.	405	-0.0014	0.0007	0.
406	0.0007	0.0013	-0.	407	0.0013	-0.0005	-0.
408	-0.0005	-0.0010	0.	409	-0.0009	0.0003	-0.
410	0.0003	0.0005	-0.	411	0.0003	-0.0002	0.
412	-0.0001	-0.	0.	413	0.0002	0.	-0.
414	-0.	-0.0003	-0.	415	-0.0004	0.0001	0.
416	0.0001	0.0003	0.	417	0.0004	-0.0001	-0.
418	-0.0001	-0.0002	-0.	419	-0.0002	0.0001	0.
420	0.0002	0.0001	0.	421	0.	-0.0002	-0.
422	-0.0003	0.	0.	423	0.	0.0003	0.
424	0.0004	0.	-0.	425	0.0001	-0.0004	-0.
426	-0.0005	-0.0001	0.	427	-0.0002	0.0005	0.
428	0.0005	0.0001	-0.	429	0.0001	-0.0004	-0.
430	-0.0003	0.	0.	431	0.0001	0.0001	0.
432	0.	-0.0003	-0.	433	-0.0004	0.	0.
434	0.0001	0.0005	-0.	435	0.0007	-0.0001	-0.
436	-0.0001	-0.0007	0.	437	-0.0007	0.0001	0.
438	0.0001	0.0006	-0.	439	0.0006	-0.0001	-0.
440	-0.0001	-0.0004	0.	441	-0.0003	0.0001	-0.
442	0.0001	0.0002	-0.	443	0.0001	-0.	0.
444	-0.	0.	0.	445	0.0002	0.	-0.
446	0.	-0.0002	-0.	447	-0.0002	0.	0.
448	-0.	0.0002	0.	449	0.0002	-0.	-0.
450	-0.	-0.0001	-0.	451	-0.0001	0.	0.
452	0.0001	0.	0.	453	0.	-0.0001	-0.
454	-0.0002	-0.	0.	455	-0.0001	0.0002	0.
456	0.0002	0.0001	-0.	457	0.0001	-0.0002	-0.
458	-0.0002	-0.0001	0.	459	-0.0001	0.0002	0.
460	0.0002	0.0001	-0.	461	0.	-0.0001	-0.
462	-0.0001	0.	0.	463	0.0001	0.0001	0.
464	0.0001	-0.0002	-0.	465	-0.0003	-0.0001	0.
466	-0.	0.0003	-0.	467	0.0003	0.0001	-0.

TABLE B-2 (CONTINUED)

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
468	0.	-0.0003	0.	469	-0.0003	-0.	0.
470	-0.	0.0003	-0.	471	0.0003	0.	-0.
472	0.	-0.0002	0.	473	-0.0001	-0.	-0.
474	0.	0.	-0.	475	-0.	-0.	0.
476	-0.	0.0001	0.	477	0.0001	0.	-0.
478	0.0001	-0.0001	-0.	479	-0.0001	-0.	0.
480	-0.	0.0001	0.	481	0.0001	0.	-0.
482	0.	-0.	-0.	483	-0.	0.	0.
484	0.0001	0.	-0.	485	0.0001	-0.0001	-0.
486	-0.0001	-0.0001	0.	487	-0.0001	0.0001	0.
488	0.0001	0.0001	-0.	489	0.0001	-0.0001	-0.
490	-0.0001	-0.0001	0.	491	-0.0001	0.0001	0.
492	0.0001	0.	-0.	493	0.	-0.0001	-0.
494	-0.0001	0.	0.	495	0.0001	0.0001	0.
496	0.0001	-0.0001	-0.	497	-0.0001	-0.0001	0.
498	-0.0001	0.0001	-0.	499	0.0002	0.0001	-0.
500	0.0001	-0.0001	-0.	501	-0.0001	-0.0001	0.
502	-0.0001	0.0001	-0.	503	0.0001	0.	-0.
504	0.	-0.	0.	505	-0.	-0.	-0.
506	0.	-0.	-0.	507	-0.	-0.	0.
508	-0.	0.	0.	509	0.	0.	0.
510	0.	-0.	-0.	511	-0.	-0.	-0.
512	-0.	0.	0.	513	0.	0.	0.
514	-0.	-0.	-0.	515	-0.	0.	0.
516	0.	0.	-0.	517	0.0001	-0.	-0.
518	-0.	-0.0001	0.	519	-0.0001	0.	0.
520	0.	0.0001	-0.	521	0.0001	-0.	-0.
522	-0.	-0.	0.	523	-0.	0.	0.
524	0.	0.	-0.	525	0.	-0.	-0.
526	-0.	0.	0.	527	0.	0.	-0.
528	0.0001	-0.	-0.	529	-0.0001	-0.0001	0.
530	-0.0001	0.0001	0.	531	0.0001	0.0001	-0.
532	0.0001	-0.0001	-0.	533	-0.	-0.	0.

Table B-3
Perturbations of Uranus.

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
0	1046.2359	114.9270	18.9247	1	-324.6979	617.3822	5.1954
2	226.6035	372.3200	19.0666	3	-222.8549	-287.1911	0.2276
4	29.9345	475.5590	23.6937	5	27.0401	-12.6566	-6.0900
6	-119.2144	61.7762	-2.7651	7	61.9505	229.2651	-2.0008
8	-33.9696	-120.3346	-7.9512	9	-7.6579	-30.9209	2.2295
10	-100.0789	10.7094	1.8721	11	41.5225	34.9671	0.9437
12	42.2824	15.5561	-0.8432	13	-36.8016	62.8247	0.7729
14	-114.7817	-29.2419	0.8517	15	23.9723	-43.1379	-1.0549
16	-21.6738	-6.0101	-0.5538	17	-6.6364	-58.5533	0.1706
18	-16.3457	1.3378	-0.4365	19	2.2163	-59.8947	1.9796
20	47.0741	0.6814	1.6342	21	2.7949	-19.1529	-1.3637
22	63.8854	7.3115	-0.1042	23	0.7623	44.4596	0.0587
24	27.5321	-4.3427	0.2121	25	-10.1897	58.6414	-0.5821
26	-55.0482	3.6993	-0.5295	27	4.6910	12.7509	0.0551
28	-56.3164	-8.4237	-0.5503	29	1.4721	-65.7747	0.4155
30	20.0343	4.5307	0.0416	31	8.5906	-34.1075	0.2489
32	69.8262	5.3215	0.5523	33	-5.5734	58.4358	-0.0430
34	-17.8546	-1.7887	0.3577	35	-10.7579	35.8151	-0.5546
36	-69.0427	-8.6662	-0.5102	37	13.2614	-75.6061	0.0780
38	50.2281	12.6758	-0.4265	39	-3.0135	11.4073	0.4193
40	35.3541	-2.2451	0.4906	41	-9.9301	70.0625	-0.0253
42	-92.6819	-14.3570	0.3888	43	14.6578	-98.0510	-0.5027
44	91.7728	17.0031	-0.7487	45	-9.9251	77.0110	0.4298
46	-61.3134	-8.6986	0.1674	47	6.3961	-44.9900	0.1228
48	32.1787	0.9389	0.6034	49	-3.7910	20.8017	-0.5630
50	-13.4623	5.1197	-0.9312	51	0.5815	-6.9423	0.6102
52	4.6220	-6.1284	0.8076	53	-1.0443	2.4794	-0.4608
54	-1.6530	2.9332	-0.5143	55	1.4863	-1.2581	0.2735
56	0.2968	-2.1083	0.2656	57	0.4782	-0.3602	-0.1070
58	0.0225	1.8881	-0.1199	59	-0.2032	0.3292	0.0383
60	0.3238	0.3054	0.0514	61	-1.0427	0.6225	-0.0227
62	-0.0633	-0.5152	-0.0240	63	-0.1228	-0.0217	-0.0078
64	-0.3701	-1.0610	0.0101	65	0.7876	-0.5975	0.0120
66	0.0366	0.4962	-0.0012	67	0.5901	-0.2446	0.0113
68	0.3176	0.9420	0.0004	69	-0.8977	0.6664	-0.0041
70	0.1491	-0.2709	-0.0028	71	-0.5198	0.1274	-0.0157
72	-0.5384	-1.0900	0.0033	73	0.8607	-0.4863	0.0040
74	0.0376	0.5400	-0.0036	75	0.5193	-0.0568	0.0130
76	0.5035	0.9592	0.0037	77	-1.1360	0.3982	-0.0012
78	-0.1965	-0.8785	-0.0032	79	-0.1257	-0.1414	-0.0151
80	-0.4837	-0.5634	0.0015	81	1.3466	-0.2246	0.0059
82	0.5815	1.1359	0.0018	83	-0.8817	0.2533	0.0123
84	0.0417	-0.1765	-0.0021	85	-0.5410	0.0493	-0.0102
86	-0.6877	-1.0182	-0.0021	87	1.3901	-0.2237	-0.0069
88	0.7842	1.2434	0.0042	89	-1.0340	-0.0076	0.0142
90	-0.4007	-0.3809	-0.0019	91	-0.1332	0.3631	-0.0036
92	-0.1293	-0.8638	-0.0010	93	1.2345	-0.5633	-0.0114
94	0.5489	1.6506	0.0028	95	-1.6640	0.6495	0.0152
96	-0.6842	-1.7151	-0.0032	97	1.5063	-0.6599	-0.0055
98	0.5466	1.3690	0.0013	99	-1.1434	0.5484	-0.0090
100	-0.3423	-0.9232	0.0013	101	0.7606	-0.3763	0.0192
102	0.2207	0.4948	-0.0020	103	-0.4113	0.2479	-0.0218
104	-0.1482	-0.1946	0.0016	105	0.1845	-0.1629	0.0192
106	0.0532	0.0854	-0.0016	107	-0.1007	0.0867	-0.0140
108	0.0254	-0.0626	0.0014	109	0.0659	-0.0366	0.0085
110	-0.0129	0.0105	-0.0007	111	-0.0205	0.0210	-0.0043
112	-0.0424	0.0390	0.0004	113	-0.0046	-0.0093	0.0022

TABLE B-3 (CONTINUED)

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
114	0.0325	-0.0187	-0.0007	115	-0.0064	-0.0070	-0.0012
116	0.0245	-0.0317	0.0006	117	0.0119	0.0070	0.0003
118	-0.0312	0.0331	-0.0001	119	0.0092	0.0092	0.0003
120	-0.0139	0.0123	-0.0001	121	-0.0199	-0.0163	-0.0002
122	0.0365	-0.0399	-0.0003	123	-0.0092	-0.0002	-0.0002
124	-0.0153	0.0223	0.0003	125	0.0485	0.0260	0.0003
126	-0.0133	0.0062	0.0001	127	-0.0496	-0.0285	0.0001
128	0.0238	-0.0093	-0.0004	129	0.0076	-0.0068	-0.0004
130	-0.0195	-0.0083	0.0002	131	0.0365	0.0511	0.0003
132	0.0160	0.0182	0.0003	133	-0.0436	-0.0621	-0.0001
134	-0.0251	-0.0061	-0.0004	135	0.0143	0.0328	-0.
136	0.0409	-0.0181	-0.	137	0.0196	0.0046	0.
138	-0.0480	0.0350	0.0006	139	-0.0320	-0.0186	-0.
140	0.0395	-0.0313	-0.0007	141	0.0202	0.0082	0.0001
142	-0.0183	0.0080	0.0004	143	0.0031	0.0093	-0.0002
144	-0.0090	0.0217	0.0002	145	-0.0257	-0.0240	0.0003
146	0.0306	-0.0419	-0.0004	147	0.0407	0.0350	-0.0003
148	-0.0362	0.0468	0.0003	149	-0.0455	-0.0381	0.0002
150	0.0295	-0.0414	0.	151	0.0401	0.0302	0.0001
152	-0.0219	0.0333	-0.0004	153	-0.0276	-0.0179	-0.0003
154	0.0182	-0.0256	0.0006	155	0.0148	0.0097	0.0003
156	-0.0148	0.0179	-0.0007	157	-0.0075	-0.0058	-0.0003
158	0.0091	-0.0100	0.0006	159	0.0058	0.0030	0.0002
160	-0.0033	0.0034	-0.0004	161	-0.0058	-0.0008	-0.0002
162	0.0004	0.0002	0.0003	163	0.0041	0.0003	0.0001
164	-0.0011	-0.0003	-0.0002	165	-0.0001	-0.0007	-0.0001
166	0.0029	-0.0020	0.0001	167	-0.0043	-0.0002	0.
168	-0.0030	0.0043	-0.0001	169	0.0062	0.0020	0.
170	0.0009	-0.0047	0.	171	-0.0044	-0.0021	-0.
172	0.0023	0.0026	-0.	173	0.0010	-0.0010	-0.
174	-0.0044	0.0005	0.	175	0.0014	0.0047	0.
176	0.0046	-0.0024	-0.	177	-0.0022	-0.0061	-0.0001
178	-0.0041	0.0026	-0.	179	0.0022	0.0049	0.
180	0.0046	-0.0018	-0.	181	-0.0021	-0.0030	0.
182	-0.0056	0.0010	0.	183	0.0015	0.0018	-0.
184	0.0050	-0.0003	-0.0001	185	-0.0003	-0.0012	0.0001
186	-0.0026	-0.0005	0.	187	-0.0009	0.0013	-0.
188	-0.	0.0009	-0.	189	0.0014	-0.0019	0.
190	0.0014	-0.0006	0.	191	-0.0010	0.0021	-0.
192	-0.0013	-0.0001	-0.	193	0.0002	-0.0015	0.
194	0.0004	0.0003	0.	195	0.0002	0.0001	-0.
196	0.0007	0.0001	-0.	197	0.	0.0012	0.
198	-0.0017	-0.0007	0.	199	-0.0004	-0.0021	-0.
200	0.0021	0.0008	-0.	201	0.0004	0.0023	0.
202	-0.0021	-0.0004	0.	203	0.0002	-0.0022	0.
204	0.0018	-0.0002	0.	205	-0.0009	0.0017	-0.
206	-0.0014	0.0005	-0.	207	0.0010	-0.0012	0.
208	0.0010	-0.0004	0.	209	-0.0003	0.0009	-0.
210	-0.0004	-0.0002	-0.	211	-0.0007	-0.0007	0.
212	0.	0.0008	0.	213	0.0011	0.0005	-0.
214	-0.0001	-0.0009	-0.	215	-0.0008	-0.0001	0.
216	0.0004	0.0006	0.	217	0.0003	-0.0002	-0.
218	-0.0006	-0.0001	-0.	219	0.0001	0.0005	0.
220	0.0007	-0.0003	0.	221	-0.0004	-0.0006	-0.
222	-0.0008	0.0005	0.	223	0.0005	0.0007	0.
224	0.0008	-0.0005	-0.	225	-0.0005	-0.0007	-0.
226	-0.0008	0.0004	0.	227	0.0004	0.0006	-0.
228	0.0007	-0.0001	-0.	229	-0.0001	-0.0005	0.
230	-0.0005	-0.0001	0.	231	-0.0002	0.0004	-0.

TABLE B-3 (CONTINUED)

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
232	0.0003	0.0002	-0.	233	0.0003	-0.0002	0.
234	-0.0002	-0.0001	0.	235	-0.0002	0.0001	-0.
236	0.0001	0.	-0.	237	0.0001	-0.0001	0.
238	-0.	0.	0.	239	0.	0.	-0.
240	0.	0.0001	-0.	241	-0.	0.	0.
242	-0.0001	-0.0002	0.	243	-0.0001	-0.	-0.
244	0.	0.0002	-0.	245	0.0001	0.	0.
246	0.	-0.0001	0.	247	0.	-0.	0.
248	-0.	-0.0001	-0.	249	-0.0001	0.	-0.
250	0.0001	0.0002	0.	251	0.0001	-0.	0.
252	-0.0001	-0.0002	-0.	253	0.0001	0.	-0.
254	0.0001	0.	-0.	255	-0.0002	-0.	-0.
256	-0.0001	0.0001	0.	257	0.0003	-0.	0.
258	0.	-0.0001	-0.	259	-0.0003	0.0001	-0.
260	0.	0.0001	0.	261	0.0002	-0.0001	0.
262	-0.0001	-0.	0.	263	-0.0001	0.0001	0.
264	0.0001	-0.	-0.	265	-0.	-0.0001	-0.
266	-0.0001	0.0001	0.	267	0.0001	0.0001	0.
268	0.0001	-0.0001	-0.	269	-0.0001	-0.0001	-0.
270	-0.0001	0.0001	0.	271	0.0001	0.0001	-0.
272	0.0001	-0.0001	-0.	273	-0.	-0.0001	0.
274	-0.0001	-0.	0.	275	-0.	0.0001	-0.
276	0.0001	0.	-0.	277	0.0001	-0.0001	0.
278	-0.0001	-0.	0.	279	-0.	0.	-0.

Table B-4
Perturbations of Neptune.

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
0	-411.6061	38.4491	-4.4141	1	30.3706	2909.4704	1.9610
2	-314.3147	19.1038	-2.7387	3	41.3092	744.1504	-0.4436
4	150.1879	-13.1648	-0.0337	5	13.4863	-109.3482	-0.7171
6	-78.8934	-19.8492	0.7382	7	-6.0134	85.4033	-0.1208
8	-52.0069	4.3401	0.5648	9	4.0535	9.3467	0.1294
10	-54.3633	7.0986	0.7918	11	6.9822	-26.7925	0.6222
12	-11.9778	10.9094	0.4336	13	-0.9346	-11.6434	0.3230
14	-2.5515	10.0030	0.1630	15	-10.6279	19.9793	-0.3597
16	-64.4477	-1.7952	0.0343	17	-4.6072	-76.0089	-0.1326
18	35.0031	0.3213	-0.2140	19	-8.7546	-15.7353	0.7920
20	21.7988	-7.8641	-0.7558	21	-0.3055	-10.5149	-0.5906
22	39.0558	-8.2485	-0.3973	23	6.3522	28.9877	0.1142
24	10.7753	-2.7634	-0.0655	25	8.9178	38.4163	-0.2327
26	-27.1551	5.7870	0.5057	27	2.6302	8.6632	0.0055
28	-37.4213	8.6528	0.5645	29	-6.9138	-33.7877	0.1297
30	1.6711	0.3931	-0.0088	31	-7.1445	-31.4725	0.1205
32	40.9017	-8.5455	-0.6152	33	4.1445	20.7807	-0.0489
34	12.6500	-2.6388	-0.3554	35	8.3732	39.1482	-0.1358
36	-40.3282	9.1504	0.5101	37	-5.1685	-20.3646	0.0088
38	-10.5962	1.6566	0.4789	39	-8.2984	-36.9087	0.1519
40	47.2941	-10.8787	-0.5386	41	9.0517	40.6530	-0.0175
42	-20.0523	4.0783	-0.3597	43	2.4648	7.1019	-0.1809
44	-32.0517	7.2589	0.7497	45	-11.2518	-50.6464	0.1292
46	59.0342	-12.9363	-0.1838	47	13.5971	58.6632	0.0600
48	-52.9570	12.8341	-0.6162	49	-10.5590	-43.2969	-0.1941
50	34.6347	-8.2015	0.9525	51	5.3232	25.0309	0.2254
52	-18.0318	3.9720	-0.8242	53	-2.7263	-11.1613	-0.1605
54	7.1232	-2.3862	0.5391	55	1.4499	3.9695	0.0834
56	-2.7924	0.7096	-0.2741	57	0.1178	-1.8865	-0.0490
58	1.3100	0.2392	0.1111	59	0.0018	0.5250	0.0189
60	0.0398	0.4518	-0.0499	61	-0.5422	0.3443	0.0020
62	-0.1158	-0.2491	0.0179	63	-0.2422	0.2444	0.0064
64	-0.4662	-0.5204	0.0035	65	0.4083	-0.3971	-0.0064
66	-0.0023	-0.0108	0.0023	67	0.4136	-0.3664	-0.0071
68	0.4723	0.5165	-0.0079	69	-0.2952	0.2742	0.0024
70	0.1193	0.1338	-0.0040	71	-0.4574	0.4554	0.0083
72	-0.4599	-0.5080	0.0067	73	0.2666	-0.2489	-0.0011
74	-0.1566	-0.1425	0.0055	75	0.4707	-0.4542	-0.0083
76	0.4854	0.5513	-0.0064	77	-0.3548	0.3053	0.0009
78	0.0650	0.0533	-0.0054	79	-0.4211	0.3758	0.0089
80	-0.4986	-0.6118	0.0072	81	0.5207	-0.4387	-0.0027
82	0.1357	0.1815	0.0035	83	0.1986	-0.1702	-0.0088
84	0.4481	0.5267	-0.0085	85	-0.6267	0.5676	0.0063
86	-0.4530	-0.5077	0.0006	87	0.2526	-0.2515	0.0056
88	-0.1089	-0.1089	0.0085	89	0.3900	-0.3722	-0.0096
90	0.5801	0.6069	-0.0069	91	-0.6653	0.6258	0.0020
92	-0.5084	-0.5705	-0.0030	93	0.3769	-0.3323	0.0077
94	0.0263	0.0557	0.0100	95	0.2278	-0.1964	-0.0107
96	0.4858	0.5366	-0.0080	97	-0.7027	0.6174	0.0047
98	-0.7753	-0.8259	0.0001	99	0.8305	-0.7790	0.0053
100	0.7909	0.7961	0.0079	101	-0.7304	0.6997	-0.0121
102	-0.6222	-0.6392	-0.0123	103	0.5565	-0.4996	0.0135
104	0.4071	0.4474	0.0124	105	-0.3538	0.3059	-0.0118
106	-0.2367	-0.2475	-0.0096	107	0.1768	-0.1720	0.0089
108	0.1244	0.1130	0.0063	109	-0.0909	0.0838	-0.0055
110	-0.0508	-0.0715	-0.0037	111	0.0672	-0.0265	0.0027
112	0.0128	0.0527	0.0020	113	-0.0333	0.0058	-0.0016

TABLE B-4 (CONTINUED)

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
114	-0.0078	-0.0106	-0.0008	115	-0.0075	-0.0092	0.0011
116	0.0093	-0.0157	0.0002	117	0.0093	0.0068	-0.0004
118	0.0005	-0.0015	-0.0002	119	0.0162	0.0044	-0.0002
120	-0.0076	0.0222	0.0001	121	-0.0183	-0.0060	0.0001
122	0.0008	-0.0084	0.	123	-0.0081	-0.0028	0.0003
124	0.0075	-0.0179	-0.0001	125	0.0222	0.0073	-0.0003
126	-0.0053	0.0175	0.	127	-0.0044	-0.0022	-0.0002
128	-0.0036	0.0075	0.0001	129	-0.0196	-0.0054	0.0003
130	0.0087	-0.0226	-0.0001	131	0.0192	0.0068	-0.
132	-0.0043	0.0096	0.	133	0.0045	-0.0002	-0.0003
134	-0.0056	0.0152	0.0001	135	-0.0246	-0.0070	0.0003
136	0.0101	-0.0242	0.	137	0.0203	0.0060	0.0001
138	-0.0039	0.0093	-0.0001	139	0.0033	0.0019	-0.0004
140	-0.0059	0.0143	0.0001	141	-0.0246	-0.0074	0.0003
142	0.0101	-0.0268	0.	143	0.0284	0.0063	0.0001
144	-0.0077	0.0210	-0.0001	145	-0.0146	-0.0015	-0.0004
146	0.0026	-0.0024	0.0001	147	-0.0067	-0.0038	0.0004
148	0.0031	-0.0181	-0.0001	149	0.0250	0.0088	-0.0002
150	-0.0083	0.0316	0.	151	-0.0341	-0.0117	-0.0002
152	0.0104	-0.0352	0.0001	153	0.0340	0.0108	0.0004
154	-0.0088	0.0311	-0.0002	155	-0.0279	-0.0077	-0.0006
156	0.0064	-0.0236	0.0002	157	0.0199	0.0056	0.0006
158	-0.0052	0.0161	-0.0001	159	-0.0129	-0.0048	-0.0004
160	0.0044	-0.0101	0.0001	161	0.0077	0.0037	0.0003
162	-0.0028	0.0056	-0.0001	163	-0.0042	-0.0016	-0.0002
164	0.0010	-0.0026	0.0001	165	0.0020	0.0003	0.0001
166	-0.0004	0.0012	0.	167	-0.0009	-0.0005	-0.0001
168	0.0008	-0.0007	0.	169	0.0004	0.0011	0.
170	-0.0010	0.0005	0.	171	-0.0001	-0.0009	-0.
172	0.0003	-0.0001	0.	173	-0.0001	-0.0003	0.
174	0.0007	-0.0002	0.	175	0.0001	0.0009	-0.
176	-0.0007	0.0002	0.	177	0.	-0.0004	-0.
178	-0.0002	-0.0001	0.	179	-0.0001	-0.0006	0.
180	0.0009	-0.0002	0.	181	0.	0.0010	-0.
182	-0.0006	0.0002	0.	183	0.0001	-0.0004	-0.
184	-0.0003	-0.0001	0.	185	-0.0002	-0.0006	0.
186	0.0010	-0.0002	0.	187	0.0002	0.0009	-0.
188	-0.0009	0.0003	0.	189	-0.0001	-0.0006	-0.
190	0.0001	-0.0001	0.	191	-0.0002	-0.0002	0.
192	0.0009	-0.0001	0.	193	0.0004	0.0009	-0.
194	-0.0013	0.0002	0.	195	-0.0004	-0.0010	-0.
196	0.0011	-0.0001	0.	197	0.0003	0.0005	0.
198	-0.0004	0.	-0.	199	-0.	0.0003	-0.
200	-0.0004	0.0001	0.	201	-0.0002	-0.0009	0.
202	0.0010	-0.0003	0.	203	0.0003	0.0013	0.
204	-0.0013	0.0004	0.	205	-0.0004	-0.0014	-0.
206	0.0013	-0.0004	0.	207	0.0003	0.0013	0.
208	-0.0011	0.0003	0.	209	-0.0002	-0.0010	-0.
210	0.0009	-0.0002	0.	211	0.0001	0.0007	0.
212	-0.0006	0.0001	0.	213	-0.0001	-0.0005	-0.
214	0.0004	-0.0001	0.	215	0.	0.0003	0.
216	-0.0002	0.	-0.	217	0.	-0.0002	-0.
218	0.0001	0.	0.	219	-0.	0.0001	0.
220	-0.0001	-0.	-0.	221	0.	-0.0001	-0.

Table B-5
Perturbations of Pluto.

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
0	-1356.4809	137.2600	13.7957	1	-20.5252	2323.0315	-29.7122
2	-786.8836	104.3215	3.1886	3	1.8086	718.7075	7.7612
4	33.6059	46.2568	-2.2112	5	9.6316	-76.4699	9.7573
6	-46.8920	17.7760	9.8055	7	-9.9343	-20.6760	0.8086
8	-38.5459	17.1075	4.8078	9	-16.6988	-44.5290	-1.4881
10	-7.9910	12.7964	5.8275	11	-11.4801	-25.4650	-8.5257
12	17.1255	1.3457	2.6179	13	-12.8121	4.2048	-6.4041
14	-3.6400	-5.8537	0.5422	15	-16.6475	-5.5124	-0.2549
16	-8.5665	-10.3037	0.4756	17	0.9194	-41.5368	-2.9302
18	57.0588	-7.4921	-0.9755	19	8.7232	41.3920	-11.6437
20	10.0370	-23.0816	-6.8766	21	15.5710	8.0754	7.5727
22	8.5368	3.9419	-2.4730	23	11.4677	23.2704	2.6544
24	-24.0420	9.0218	-0.7115	25	-3.8443	-4.6200	6.7260
26	-28.4132	15.9775	3.9995	27	-14.1639	-23.7016	0.4020
28	-2.8746	3.1190	4.4312	29	-7.9682	-17.9171	-5.5097
30	28.2363	-12.4906	0.0679	31	8.6156	13.9739	-5.3640
32	19.9103	-10.9930	-4.6647	33	11.0463	27.6193	2.3236
34	-21.1341	7.1247	-2.7161	35	-5.7658	-5.6023	6.8764
36	-25.1819	11.8971	3.8666	37	-10.7807	-30.9878	-0.2089
38	21.3229	-6.1771	3.5694	39	7.2115	9.0028	-7.1302
40	20.2684	-9.7995	-4.2024	41	7.0107	31.5954	1.0210
42	-32.4938	8.6704	-2.8346	43	-9.8840	-30.8191	7.1690
44	4.7307	3.9612	5.8178	45	2.3855	-8.3005	-5.8522
46	28.5097	-9.6662	-1.3034	47	5.7622	43.4454	-2.5336
48	-39.5294	6.2570	-4.6585	49	-9.4379	-51.0422	8.9817
50	31.8347	-0.8494	7.3142	51	7.7627	40.1719	-9.6082
52	-19.9073	-2.6644	-6.4331	53	-4.7254	-24.5721	7.1400
54	9.5974	2.9632	4.0460	55	2.8862	11.5953	-4.1812
56	-3.4806	-1.6542	-2.0991	57	-1.1122	-4.9597	1.8040
58	1.6920	1.1132	0.9617	59	0.1066	2.3035	-0.7441
60	-0.6675	-0.6471	-0.2951	61	-0.4028	-0.3986	0.4035
62	-0.3055	-0.1027	0.0876	63	0.2052	-0.0994	-0.0237
64	-0.0744	-0.0072	-0.0934	65	0.3465	-0.4466	-0.0641
66	0.4361	0.3349	-0.0039	67	-0.0017	0.1382	-0.1242
68	0.1974	0.0652	0.0519	69	-0.3590	0.4365	0.0461
70	-0.3669	-0.3139	0.0325	71	-0.0689	-0.0192	0.1334
72	-0.2666	-0.1269	-0.0453	73	0.3553	-0.4526	-0.0147
74	0.3477	0.3368	-0.0365	75	0.0650	0.0073	-0.1394
76	0.2476	0.1322	0.0458	77	-0.3925	0.4554	0.0127
78	-0.3806	-0.3991	0.0361	79	0.0157	-0.0876	0.1419
80	-0.1387	-0.0669	-0.0569	81	0.4232	-0.4219	-0.0407
82	0.4212	0.4665	-0.0282	83	-0.1855	0.2463	-0.1319
84	-0.0614	-0.1299	0.0717	85	-0.3330	0.2948	0.0953
86	-0.3864	-0.4118	-0.0017	87	0.3876	-0.4426	0.0845
88	0.3367	0.4269	-0.0669	89	0.0114	0.0554	-0.1539
90	0.0993	0.0554	0.0471	91	-0.3876	0.3942	0.0356
92	-0.4208	-0.5214	0.0215	93	0.4283	-0.4648	0.1247
94	0.3667	0.5439	-0.0709	95	-0.1173	0.1503	-0.1684
96	-0.0451	-0.0995	0.0615	97	-0.3023	0.2569	0.0721
98	-0.3025	-0.4543	0.0023	99	0.5389	-0.5107	0.0775
100	0.4970	0.7752	-0.0676	101	-0.5420	0.5576	-0.1858
102	-0.5010	-0.8224	0.0923	103	0.4453	-0.4613	0.2169
104	0.3921	0.7097	-0.0867	105	-0.3232	0.3081	-0.1893
106	-0.2624	-0.5131	0.0728	107	0.1861	-0.1720	0.1366
108	0.1556	0.3021	-0.0516	109	-0.0825	0.0874	-0.0840
110	-0.0780	-0.1608	0.0266	111	0.0483	-0.0394	0.0454
112	0.0300	0.0995	-0.0130	113	-0.0409	0.0083	-0.0226

TABLE B-5 (CONTINUED)

K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	K	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$
114	-0.0120	-0.0569	0.0117	115	0.0139	0.0034	0.0103
116	0.0099	0.0099	-0.0084	117	0.0115	0.0013	-0.0039
118	-0.0041	0.0080	-0.0002	119	-0.0039	-0.0035	0.0011
120	-0.0043	0.0084	0.0031	121	-0.0154	-0.0024	-0.0003
122	0.0033	-0.0183	0.0020	123	0.0110	0.0050	0.0005
124	0.0038	0.0014	-0.0047	125	0.0097	0.0002	-0.0005
126	-0.0053	0.0168	0.0001	127	-0.0157	-0.0056	-0.0001
128	0.	-0.0108	0.0046	129	-0.0001	0.0036	0.0005
130	0.0050	-0.0101	-0.0028	131	0.0155	0.0039	0.
132	-0.0044	0.0185	-0.0028	133	-0.0119	-0.0075	-0.0005
134	-0.0012	-0.0044	0.0049	135	-0.0058	0.0018	0.0002
136	0.0060	-0.0157	-0.0012	137	0.0177	0.0068	0.0003
138	-0.0044	0.0199	-0.0041	139	-0.0120	-0.0080	-0.0005
140	-0.0021	-0.0051	0.0052	141	-0.0047	0.0009	0.0004
142	0.0066	-0.0142	-0.0012	143	0.0175	0.0071	-0.0001
144	-0.0055	0.0229	-0.0039	145	-0.0178	-0.0097	-0.0005
146	0.0014	-0.0168	0.0060	147	0.0075	0.0066	0.0008
148	0.0026	0.0012	-0.0043	149	0.0065	0.0002	-0.0004
150	-0.0060	0.0155	0.0002	151	-0.0177	-0.0084	-0.0002
152	0.0079	-0.0263	0.0041	153	0.0228	0.0142	0.0006
154	-0.0073	0.0288	-0.0069	155	-0.0218	-0.0152	-0.0007
156	0.0049	-0.0252	0.0076	157	0.0173	0.0131	0.0008
158	-0.0032	0.0192	-0.0066	159	-0.0121	-0.0105	-0.0008
160	0.0028	-0.0132	0.0049	161	0.0080	0.0083	0.0005
162	-0.0023	0.0081	-0.0034	163	-0.0048	-0.0058	-0.0001
164	0.0011	-0.0043	0.0022	165	0.0025	0.0030	-0.0001
166	-0.0001	0.0020	-0.0012	167	-0.0010	-0.0013	0.
168	0.0001	-0.0010	0.0006	169	0.0005	0.0010	0.0001
170	-0.0007	0.0006	-0.0003	171	-0.0004	-0.0012	0.0001
172	0.0007	-0.0002	0.0002	173	0.0002	0.0007	-0.0002
174	0.	-0.0001	-0.0001	175	0.	0.0002	0.0001
176	-0.0006	0.0002	0.	177	-0.0002	-0.0007	0.0001
178	0.0004	0.	0.0001	179	0.0002	0.0002	-0.0001
180	0.0003	-0.0001	-0.0001	181	0.	0.0006	0.
182	-0.0007	0.0001	0.	183	-0.0002	-0.0007	0.0001
184	0.0004	0.	0.0001	185	0.0001	0.0002	-0.0002
186	0.0003	-0.0002	-0.0001	187	0.	0.0006	0.
188	-0.0007	0.0002	0.	189	-0.0002	-0.0008	0.0001
190	0.0005	-0.0001	0.0001	191	0.0002	0.0004	-0.0002
192	0.	-0.0001	-0.0001	193	0.	0.0003	0.0001
194	-0.0006	0.0002	0.	195	-0.0001	-0.0009	0.0001
196	0.0008	-0.0001	0.0001	197	0.0002	0.0010	-0.0002
198	-0.0006	0.	-0.0001	199	-0.0002	-0.0006	0.0002
200	0.0001	0.0001	0.0001	201	0.0001	-0.0001	-0.0001
202	0.0004	-0.0001	0.	203	0.0001	0.0008	0.
204	-0.0008	0.0001	-0.0001	205	-0.0002	-0.0011	0.0002
206	0.0009	-0.0001	0.0001	207	0.0002	0.0012	-0.0002
208	-0.0008	0.0001	-0.0002	209	-0.0002	-0.0012	0.0002
210	0.0007	0.	0.0002	211	0.0001	0.0010	-0.0002
212	-0.0005	-0.0001	-0.0001	213	-0.0001	-0.0007	0.0002
214	0.0004	0.0001	0.0001	215	0.0001	0.0005	-0.0001
216	-0.0002	-0.0001	-0.0001	217	0.	-0.0003	0.0001
218	0.0001	0.	0.	219	0.	0.0002	-0.
220	-0.0001	-0.0001	0.	221	0.	-0.0001	0.
222	0.0001	0.0001	0.	223	0.	0.0001	-0.

Appendix C

Comparisons with Tabulated Coordinates

The perturbations computed by the CHBY program have been compared with values computed from coordinates tabulated* for each of the five outer planets. The comparisons were made at the standard 400-day dates from Julian Date 2378400.5 through Julian Date 2452000.5.

The values of the perturbations and the residuals are given at each date for each of the planets. The residuals are in all cases less than 0''.001.

*Eckert, W. J., Brouwer, D., and Clemence, G. M., "Coordinates of the Five Outer Planets 1653-2060," Astron. Papers prepared for the use of the American Ephemeris and Nautical Almanac, 12:1-327, 1951.

Table C-1
Residuals for Jupiter.

Julian Day	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	$\alpha_T \cdot 10^6$	$\beta_T \cdot 10^6$	$\gamma_T \cdot 10^6$	$(\alpha - \alpha_T) \cdot 10^6$	$(\beta - \beta_T) \cdot 10^6$	$(\gamma - \gamma_T) \cdot 10^6$
2378400.5	-732.0311	-512.872	74.4260	732.8807	-512.8871	74.4267	0.0004	-0.0000	-0.0007
2378800.5	437.3597	-1199.4720	90.0117	437.3597	-1199.4723	90.0118	0.0000	0.0003	-0.0000
2379200.5	87.0232	-1418.9492	77.6015	87.0234	-1418.9501	77.6012	-0.0001	0.0009	0.0003
2379600.5	-75.2446	-1291.1208	44.9406	-75.2453	-1291.1218	44.9402	0.0007	0.0010	0.0004
2380000.5	-26.5976	-1155.6404	3.2619	-26.5978	-1155.6404	3.2622	0.0002	-0.0000	-0.0003
2380400.5	89.4696	-1194.7152	-36.8661	89.4704	-1194.7151	-36.8658	-0.0008	-0.0000	-0.0003
2380800.5	149.4111	-1381.3367	-64.8959	149.4113	-1381.3382	-64.8959	-0.0002	0.0006	0.0000
2381200.5	96.7760	-1585.9291	-71.8973	96.7754	-1585.9294	-71.8974	0.0007	0.0003	0.0001
2381600.5	-57.6336	-1662.2433	-54.2094	-57.0627	-1662.2443	-54.2092	-0.0009	0.0009	-0.0002
2382000.5	-42.1022	-1521.8202	-16.9138	-242.1021	-1521.8207	-16.9145	-0.0001	0.0005	0.0008
2382400.5	-65.8214	-1182.0546	26.4606	-365.8217	-1182.0560	26.4610	0.0002	0.0114	-0.0004
2382800.5	-366.5223	-759.5120	59.9786	-366.5226	-759.5121	59.9786	0.0003	0.0001	0.0002
2383200.5	-243.9145	-410.0809	72.7798	-243.9146	-410.0820	72.7799	0.0001	0.0011	-0.0002
2383600.5	-49.5765	-257.5708	62.3355	-49.5769	-257.5009	62.3359	0.0004	0.0001	-0.0004
2384000.5	141.4346	-347.3736	33.2438	141.4337	-347.3734	33.2435	0.0009	-0.0002	0.0003
2384400.5	255.3172	-634.3332	-55.4214	255.9176	-634.3831	-54.4222	-0.0004	-0.0001	0.0007
2384800.5	240.0113	-994.9495	-42.4429	240.8118	-994.9507	-42.4427	-0.0001	0.0012	-0.0002
2385200.5	79.3696	-1258.9484	-66.5642	79.3693	-1258.9487	-66.5643	0.0002	-0.0000	0.0001
2385600.5	-189.4467	-1259.9798	-69.2370	-189.5459	-1259.9805	-69.2372	-0.0008	0.0006	0.0002
2386000.5	-449.1765	-909.7491	-48.5002	-449.1762	-909.7492	-48.5004	-0.0003	0.0001	0.0002
2386400.5	-07.0204	-308.5256	-12.4580	-507.8207	-308.5254	-12.4581	0.0003	-0.0002	0.0001
2386800.5	-50.5059	178.9359	23.5305	-250.0570	178.9361	23.5301	0.0011	-0.0003	0.0004
2387200.5	151.3505	205.3440	47.4070	151.3501	205.3440	47.4067	0.0004	0.0000	0.0003
2387600.5	441.8550	-231.2335	53.8548	441.8548	-231.2329	53.8547	0.0002	-0.0006	0.0001
2388000.5	489.1706	-886.0155	43.0891	489.1715	-886.0054	43.0891	-0.0009	-0.0001	0.0000
2388400.5	-85.0100	-1444.3409	19.6467	285.6095	-1444.3412	19.6464	0.0005	0.0002	0.0003
2388800.5	-90.4632	-1643.6691	-9.2904	-90.4638	-1643.6688	-9.2902	0.0006	-0.0003	-0.0002
2389200.5	-08.3713	-1350.8128	-35.3844	-508.3707	-1350.8129	-35.3843	-0.0006	0.0000	-0.0001
2389600.5	-15.7704	-604.4116	-50.6107	-815.7699	-604.4120	-50.6109	-0.0005	0.0003	0.0002
2390000.5	-379.2857	381.759	49.2829	-879.2854	381.7056	-49.2822	-0.0003	0.0003	-0.0007
2390400.5	-42.1525	1273.3997	-30.7222	-642.1526	1273.4004	-30.7221	0.0002	-0.0007	-0.0001
2390800.5	-173.9270	1740.4459	-1.0290	-173.9271	1740.4466	-1.0293	0.0001	-0.0007	0.0003
2391200.5	442.3469	1606.8469	28.7518	342.8469	1606.8471	28.7517	0.0001	-0.0003	0.0001
2391600.5	702.4345	935.5527	47.9649	702.4342	935.5518	47.9649	0.0003	0.0009	0.0000
2392000.5	772.7498	-45.5313	50.9496	772.7493	-45.5311	50.9495	0.0005	-0.0002	0.0000
2392400.5	33.2722	-851.1534	38.0232	533.2710	-851.1526	38.0237	0.0012	-0.0008	-0.0005
2392800.5	62.6270	-1271.7478	14.0294	62.6263	-1271.7477	14.0296	0.0007	-0.0001	-0.0002
2393200.5	-185.2577	-1062.8795	-13.4872	-485.2575	-1062.8802	-13.4877	-0.0002	0.0007	0.0005
2393600.5	-98.5142	-231.3963	-35.7916	-898.5146	-231.3964	-35.7915	0.0004	0.0001	-0.0001
2394000.5	-951.7063	928.2630	-44.9615	-951.7072	928.2625	-44.9612	0.0009	0.0005	-0.0003
2394400.5	-73.8695	1879.6220	-37.4598	-573.8702	1879.6219	-37.4597	0.0007	0.0001	-0.0000
2394800.5	45.7286	2187.1358	-16.0254	45.7283	2187.1357	-16.0257	0.0003	0.0002	0.0002
2395200.5	-16.9709	1756.3601	11.4376	616.9706	1756.3594	11.4372	0.0003	0.0006	0.0004
2395600.5	-09.5937	808.5729	34.7062	909.5933	808.5728	34.7065	0.0003	0.0001	-0.0003
2396000.5	424.9357	-259.3645	45.8448	842.9360	-259.3643	45.8447	-0.0003	-0.0002	0.0001
2396400.5	485.6692	-1055.4474	42.1382	485.6684	-1055.4871	42.1378	0.0008	-0.0003	0.0004
2396800.5	-9.9879	-1327.7119	25.8442	-9.9881	-1327.7017	25.8442	0.0003	-0.0002	-0.0000
2397200.5	-475.4886	-1018.0177	2.4121	-475.4897	-1018.0077	2.4125	0.0011	-0.0000	-0.0004
2397600.5	-768.6615	-253.6787	-21.2458	-768.6609	-253.6783	-21.2451	-0.0005	-0.0004	-0.0006
2398000.5	-100.3308	700.3353	-38.1306	-800.3307	700.3352	-38.1314	-0.0001	0.0001	0.0008
2398400.5	-557.5024	1517.6985	-42.6191	-557.5023	1517.6984	-42.6195	-0.0001	0.0001	0.0005
2398800.5	-122.6036	1913.3668	-32.3661	-122.6036	1913.3674	-32.3659	0.0000	-0.0006	-0.0002
2399200.5	33.2562	1756.8238	-10.1913	333.2560	1756.8944	-10.1914	0.0002	-0.0006	0.0001
2399600.5	613.8895	1146.2123	15.9224	613.8899	1146.2128	15.9229	-0.0004	-0.0005	-0.0005
2400000.5	-04.1262	377.2926	36.2059	604.1257	377.2931	36.2063	0.0004	-0.0005	-0.0004
2400400.5	339.9149	-195.3375	43.9085	339.9146	195.3378	43.9086	0.0003	0.0003	-0.0001
2400800.5	-1.2222	-337.2810	38.0795	-1.2228	-337.2813	38.0797	0.0006	0.0003	-0.0002
2401200.5	-177.5922	-98.1962	23.5037	-177.5918	-98.1963	23.5033	-0.0004	0.0001	0.0004
2401600.5	-80.0964	170.9213	6.8369	-80.0968	170.9209	6.8377	0.0004	0.0004	-0.0008
2402000.5	161.8848	160.9273	-8.1934	161.8854	160.9281	-8.1935	-0.0006	-0.0008	0.0001
2402400.5	57.2224	-170.7583	-19.1521	367.2222	-170.7593	-19.1514	0.0002	0.0016	-0.0006
2402800.5	415.4001	-676.7396	-23.5386	415.4006	-676.7391	-23.5386	-0.0005	-0.0004	-0.0001
2403200.5	68.4997	-1125.4782	-19.8993	268.5004	-1125.4081	-19.8998	-0.0007	-0.0001	0.0005
2403600.5	-18.7783	-1303.8734	-9.2346	-18.7776	-1303.8739	-9.2347	-0.0007	0.0005	0.0001
2404000.5	-320.5842	-1115.2404	4.6258	-320.5840	-1115.2409	4.6258	-0.0002	0.0004	0.0000
2404400.5	-05.7803	-624.2082	16.5454	-505.7797	-624.2076	16.5450	-0.0005	-0.0006	0.0005
2404800.5	-004.3904	-19.9971	22.5307	-504.5899	-19.9970	22.5315	-0.0005	-0.0001	-0.0007
2405200.5	-328.9412	473.4081	21.1345	-328.9411	473.4083	21.1342	-0.0001	-0.0002	0.0002
2405600.5	-51.9200	685.0489	13.3136	-51.9205	685.0499	13.3139	0.0005	-0.0010	-0.0003
2406000.5	725.0753	551.1350	1.6561	225.0759	551.1363	1.6562	-0.0006	-0.0013	-0.0001
2406400.5	400.7459	131.4048	-10.3983	400.7467	131.4846	-10.3982	-0.0008	0.0001	-0.0001
2406800.5	398.6657	-404.2277	-19.2360	398.6661	-404.2275	-19.2365	-0.0004	-0.0002	0.0005
2407200.5	197.0717	-819.8519	-21.9299	197.0725	-819.8514	-21.9291	-0.0008	-0.0005	-0.0008
2407600.5	-134.3066	-894.2671	-17.5182	-134.3070	-894.2679	-17.5185	0.0004	0.0007	0.0002
2408000.5	-410.1950	-548.9420	-8.5642	-410.1938	-548.9420	-8.5647	-0.0011	-0.0000	0.0005
2408400.5	-407.4762	0.6043	-0.8902	-407.4757	0.6045	-0.8909	-0.0005	-0.0002	0.0007
2408800.5	-136.3838	334.4964	2.3244	-136.3827	334.4970	2.3248	-0.0011	-0.0007	-0.0004
2409200.5	172.6299	254.7441	2.4253	172.6311	254.7448	2.4252	-0.0012	-0.0008	0.0001
2409600.5	341.4530	-141.3245	1.0380	341.4535	-141.3243	1.0374	-0.0005	-0.0002	0.0007
2410000.5	306.3694	-621.1565	-0.7802	306.3715	-621.1561	-0.7806	-0.0021	-0.0008	0.0004
2410400.5	90.7363	-941.732	-2.3230	90.7361	-941.7026	-2.3241	0.0002	-0.0006	0.0011
2410800.5	-224.6649	-929.6609	-3.1437	-224.6654	-929.6598	-3.1441	0.0005	-0.0011	0.0004
2411200.5	-524.5407	-531.9204	-3.0354	-524.5407	-531.9196	-3.0349	0.0001	-0.0008	-0.0005
2411600.5	-86.2709	159.5518	-2.0646	-86.2709	159.5526	-2.0647	-0.0001	-0.0008	0.0001
2412000.5	-623.3461	922.8263	-0.5966	-623.3466	922.8266	-0.5968	0.0005	-0.0003	0.0001
2412400.5	-336.6506	1479.7064	0.7866	-336.6502	1479.7066	0.7862	-0.0004	-0.0003	0.0004
2412800.5	67.0133	1615.9434	1.5203	67.0129	1615.9442	1.5199	0.0003	-0.0008	0.0005
2413200.5	423.2670	1284.2925	1.3376	423.2670	1284.2836	1.3377	0.0001	-0.0012	-0.0001
2413600.5	592.4803	624.4515	0.3746	592.4799	624.4526	0.3742	0.0005	-0.0011	0.00

Table C-1 (Continued)

Julian Day	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	$\alpha_T \cdot 10^6$	$\beta_T \cdot 10^6$	$\gamma_T \cdot 10^6$	$(\alpha - \alpha_T) \cdot 10^6$	$(\beta - \beta_T) \cdot 10^6$	$(\gamma - \gamma_T) \cdot 10^6$
2414800.5	-25.6802	-639.2694	-2.2820	-225.6803	-639.2894	-2.2821	0.0002	0.0001	0.0001
2415200.5	-24.7392	-146.1654	-1.1039	-624.7391	-146.1461	-1.1036	-0.0001	0.0007	-0.0003
2415600.5	-783.5787	736.6690	1.8615	-783.0788	736.6702	1.8618	0.0001	-0.0012	-0.0003
2416000.5	-66.1703	1596.2492	5.7091	-566.1697	1596.2495	5.7095	-0.0006	-0.0003	-0.0003
2416400.5	-78.7769	1987.7979	8.1031	-78.0778	1987.7978	8.1028	0.0009	0.0000	0.0003
2416800.5	-23.4569	1751.2134	7.4991	423.4513	1751.2037	7.4993	-0.0004	-0.0003	0.0002
2417200.5	17.2597	1031.675	4.0516	717.2512	1031.6083	4.0510	-0.0005	-0.0008	0.0006
2417600.5	710.3397	152.0554	-0.7638	710.9397	152.0056	-0.7833	-0.0000	-0.0001	-0.0005
2418000.5	47.3482	-547.9399	-5.1782	447.0486	-547.9492	-5.1785	-0.0004	-0.0007	0.0003
2418400.5	48.3255	-836.2413	-7.6745	48.3253	-836.2217	-7.6743	0.0002	0.0003	-0.0002
2418800.5	-42.3257	-642.6141	-7.5248	-342.3253	-642.6136	-7.5243	-0.0004	-0.0005	-0.0005
2419200.5	-0.00.8200	-56.4428	-4.7524	-600.8211	-56.4632	-4.7523	0.0011	0.0004	-0.0001
2419600.5	-45.5380	710.3124	-0.1107	-645.5383	710.3132	-0.1108	0.0003	-0.0009	0.0002
2420000.5	-57.1712	1384.7009	5.0121	-457.1713	1384.7917	5.0120	0.0001	-0.0008	0.0001
2420400.5	-96.966	1717.7230	8.8405	-96.9692	1717.7235	8.8408	0.0006	-0.0005	-0.0003
2420800.5	93.932	1582.4619	9.7777	293.0936	1582.4617	9.7777	-0.0004	0.0002	-0.0003
2421200.5	38.7410	1948.9721	7.2067	533.7412	1048.9717	7.2061	-0.0001	0.0002	0.0006
2421600.5	526.7024	370.7643	1.9726	526.7038	370.7643	1.9731	-0.0013	-0.0000	-0.0005
2422000.5	74.5992	-130.6422	-3.9351	274.5996	-130.6427	-3.9352	-0.0004	0.0005	0.0001
2422400.5	-71.8823	-221.7331	-8.0445	-71.9823	-221.7830	-8.0445	-0.0000	-0.0001	-0.0000
2422800.5	-90.8993	103.9165	-7.4863	-290.9005	103.9161	-7.4866	0.0013	0.0004	0.0003
2423200.5	-09.2133	542.3295	0.0606	-208.2140	542.3295	0.0613	0.0007	-0.0000	-0.0007
2423600.5	114.9756	678.9611	12.3534	114.9747	678.9613	12.3532	0.0009	-0.0001	0.0002
2424000.5	465.1798	347.4986	23.4665	465.1798	347.4992	23.4661	0.0001	-0.0005	0.0004
2424400.5	-56.4111	-341.1116	28.1260	656.0409	-341.1810	28.1265	0.0002	-0.0006	-0.0005
2424800.5	87.2615	-1111.4252	23.4966	587.2614	-1111.4257	23.4969	0.0000	0.0005	-0.0003
2425200.5	73.4859	-1646.2730	10.2950	273.9853	-1646.2030	10.2949	0.0006	0.0001	0.0001
2425600.5	-153.9846	-1723.5112	-6.9785	-153.9856	-1723.5521	-6.9785	0.0010	0.0009	0.0000
2426000.5	-16.4466	-1315.4557	-21.9333	-516.4485	-1315.4961	-21.9335	-0.0001	0.0004	0.0002
2426400.5	-76.0916	-589.7414	-29.4286	-676.0918	-589.7421	-29.4290	0.0002	0.0007	0.0004
2426800.5	-90.0024	177.7178	-27.5024	-590.6026	177.7577	-27.5021	0.0002	0.0001	-0.0003
2427200.5	-06.7518	715.8424	-17.2988	-308.7517	715.8623	-17.2987	-0.0001	0.0001	-0.0001
2427600.5	71.4041	845.889	-2.0662	71.4040	845.8088	-2.0666	0.0000	0.0001	0.0004
2428000.5	-14.5603	529.9118	13.8310	414.5610	529.9815	13.8310	-0.0007	0.0003	-0.0000
2428400.5	-97.2987	-114.7449	25.7247	597.5999	-114.7555	25.7244	-0.0013	0.0006	0.0003
2428800.5	35.2362	-839.9619	29.5463	535.2360	-839.9623	29.5461	0.0003	0.0004	0.0002
2429200.5	30.7956	-1336.0195	23.0649	230.7962	-1336.0192	23.0646	-0.0005	-0.0003	0.0003
2429600.5	-175.152	-1363.3134	6.9637	-175.0165	-1363.3125	6.9635	0.0013	-0.0009	0.0003
2430000.5	-18.3350	-939.6236	-15.4240	-418.3352	-939.6238	-15.4232	0.0003	0.0002	-0.0008
2430400.5	-38.5834	-432.7.33	-37.6416	-338.5835	-432.7031	-37.6414	0.0001	-0.0002	-0.0002
2430800.5	-78.6462	-205.4111	-50.3056	-78.8641	-205.4813	-50.3052	-0.0001	0.0002	-0.0004
2431200.5	155.1444	-323.7299	-68.0549	155.1443	-323.7292	-68.0554	0.0000	-0.0007	0.0005
2431600.5	-49.1694	-654.6624	-31.6816	249.1698	-654.6621	-31.6822	-0.0004	-0.0003	0.0005
2432000.5	175.5120	-994.5271	-6.1834	175.5126	-994.5274	-6.1832	-0.0006	0.0003	-0.0001
2432400.5	-30.1787	-1154.2781	21.0956	-30.1786	-1154.2793	21.0953	-0.0001	0.0012	0.0004
2432800.5	-87.2119	-1019.6450	41.9684	-287.5225	-1019.6451	41.9685	0.0006	0.0002	-0.0000
2433200.5	-91.4810	-590.8286	49.2209	-49.1480	-590.8292	49.2206	-0.0003	0.0005	0.0003
2433600.5	-45.3021	6.8722	39.1995	-545.0816	6.8720	39.1995	-0.0006	0.0002	0.0000
2434000.5	-46.1775	559.9678	14.4549	-406.1769	559.9882	14.4550	-0.0005	-0.0003	-0.0001
2434400.5	-121.4645	855.7129	-16.1167	-121.4645	855.7124	-16.1168	0.0000	0.0006	0.0000
2434800.5	191.1930	782.9228	-41.2660	191.1933	782.9224	-41.2659	-0.0003	0.0004	-0.0002
2435200.5	404.8913	381.7711	-52.7262	404.8908	381.7700	-52.7258	0.0005	0.0011	-0.0003
2435600.5	437.6705	-179.5130	-47.7967	437.6708	-179.5325	-47.7969	-0.0003	-0.0005	0.0002
2436000.5	572.5486	-669.5996	-28.0508	272.5481	-669.5995	-28.0507	0.0004	-0.0001	-0.0001
2436400.5	-42.2741	-868.4709	-1.4432	-42.2741	-868.4711	-1.4430	0.0000	0.0002	-0.0002
2436800.5	-96.3537	-640.9559	26.7676	-396.3540	-640.9854	26.7676	0.0003	-0.0005	-0.0001
2437200.5	-20.3867	-15.2181	47.8010	-620.3879	-15.2180	47.8009	0.0011	0.0007	0.0001
2437600.5	-42.8711	735.8914	54.4573	-542.8717	735.8907	54.4571	0.0006	0.0007	0.0003
2438000.5	-175.0702	1189.6778	42.5479	-175.0708	1189.6775	42.5484	0.0007	0.0003	-0.0005
2438400.5	268.1695	1113.2112	14.8416	268.1693	1113.2911	14.8413	0.0002	0.0001	0.0003
2438800.5	566.1230	575.7524	-18.5628	566.1232	575.7261	-18.5627	-0.0003	-0.0008	-0.0001
2439200.5	0.08.3608	-159.7334	-45.3184	608.0611	-159.7233	-45.3177	-0.0003	-0.0001	-0.0006
2439600.5	-11.7172	-786.4769	-56.6869	411.7176	-786.4076	-56.6866	-0.0004	0.0008	-0.0003
2440000.5	79.2731	-1079.0725	-50.0594	79.2731	-1079.0730	-50.0605	-0.0000	0.0005	0.0011
2440400.5	-60.1260	-955.6493	-28.2871	-260.1259	-955.6946	-28.2874	-0.0002	0.0003	0.0003
2440800.5	-491.0323	-482.7555	2.0486	-491.0325	-482.7062	2.0485	0.0003	0.0007	0.0002
2441200.5	-39.1502	159.3476	32.2180	-539.1506	159.3476	32.2183	0.0004	-0.0000	-0.0003
2441600.5	-82.2474	732.0267	52.9310	-382.2474	732.0272	52.9319	-0.0000	-0.0005	-0.0009
2442000.5	-70.8370	1013.7109	56.5952	-70.8369	1013.7122	56.5952	-0.0001	-0.0013	-0.0000
2442400.5	-74.0264	885.6240	40.5154	274.0247	885.6236	40.5154	-0.0000	0.0005	-0.0001
2442800.5	-97.4931	398.6068	9.5358	497.4933	398.6869	9.5360	-0.0003	-0.0001	-0.0002
2443200.5	491.0603	-229.1218	-25.0205	491.0610	-229.1218	-25.0199	-0.0007	0.0000	-0.0005
2443600.5	754.5699	-705.0881	-50.6910	254.5695	-705.0087	-50.6910	0.0005	0.0006	0.0001
2444000.5	-95.7727	-795.8733	-59.1829	-95.7718	-795.8736	-59.1826	-0.0009	0.0003	-0.0003
2444400.5	-67.5445	-446.5808	-47.7874	-367.5449	-446.5081	-47.7874	0.0004	0.0001	-0.0001
2444800.5	-59.7982	125.9686	-18.1856	-359.7984	125.9686	-18.1856	0.0002	0.0000	-0.0000
2445200.5	-33.9768	479.0556	22.1906	-33.9784	479.0860	22.1903	-0.0004	-0.0003	0.0003
2445600.5	420.0828	298.8367	59.5681	420.0829	298.8359	59.5685	-0.0002	0.0007	-0.0004
2446000.5	767.0008	-406.0676	79.9046	767.0006	-406.0684	79.9048	0.0002	0.0008	-0.0001
2446400.5	-58.4994	-1379.8652	74.0576	838.4993	-1379.8658	74.0582	0.0001	0.0006	-0.0005
2446800.5	-85.0185	-2244.7087	41.7828	589.0181	-2244.7095	41.7828	0.0004	0.0009	-0.0000
2447200.5	104.6147	-2658.6963	-6.2668	104.6155	-2658.6960	-6.2666	-0.0008	-0.0003	-0.0002
2447600.5	-400.2457	-2468.3443	-52.2534	-400.2449	-2468.3444	-52.2531	-0.0007	0.0001	-0.0003
2448000.5	-731.4179	-1768.5588	-80.0258	-731.4182	-1768.5583	-80.0253	0.0003	-0.0005	-0.0005
2448400.5	-783.4196	-839.3404	-81.5994	-783.4198	-839.3402	-81.5986	0.0002	-0.0001	-0.0008
2448800.5	-61.4336	-18.9081	-58.2079	-561.4335	-18.9083	-58.2071	-0.0001	0.0001	-0.0008
2449200.5	-154.5013	413.9894	-17.8037	-154.5001	413.9888	-17.8040	-0.0011	0.0006	0.0002
2449600.5	-299.5275	318.4330	27.8027	299.5275	318.4326	27.8025	0.0000	0.0003	0.0002
2450000.5	646.1689	-270.0801	65.3273	646.1688	-270.0798	65.3280	0.0001	-0.0004	-0.0007
2450400.5	751.2086	-1140.0829	82.5641	751.2091	-1140.0827	82.5642	-0.0005	-0.0002	-0.0001
2450800.5	52.9113	-1954.3655							

Table C-2
Residuals for Saturn.

Julian Day	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	$\alpha_T \cdot 10^6$	$\beta_T \cdot 10^6$	$\gamma_T \cdot 10^6$	$(\alpha - \alpha_T) \cdot 10^6$	$(\beta - \beta_T) \cdot 10^6$	$(\gamma - \gamma_T) \cdot 10^6$
2378400.5	-02.4108	2423.7445	-209.8672	-502.9095	2423.7669	-209.8671	-0.0012	-0.0025	-0.0001
2378800.5	52.1353	3034.9301	-198.3214	52.1358	3034.9518	-198.3214	-0.0005	-0.0018	-0.0000
2379200.5	18.8134	3389.5163	-174.7261	518.8136	3389.5176	-174.7265	-0.0003	-0.0014	0.0004
2379600.5	746.9072	3559.3115	-142.4404	746.9068	3559.3333	-142.4408	0.0004	-0.0019	0.0004
2380000.5	748.1944	3706.3177	-104.9412	748.0952	3706.3199	-104.9412	-0.0007	-0.0022	-0.0000
2380400.5	-52.1004	3885.9524	-62.2531	652.1001	3885.9538	-62.2531	0.0003	-0.0014	-0.0000
2380800.5	546.6650	4056.0167	-14.8480	546.6659	4056.0072	-14.8476	-0.0010	-0.0005	-0.0003
2381200.5	467.0167	4149.5120	34.7994	467.8176	4149.5123	34.7995	-0.0009	-0.0004	-0.0000
2381600.5	428.0454	4107.734	82.7459	428.0455	4107.7849	82.7461	-0.0001	-0.0015	-0.0002
2382000.5	432.0224	3900.115	124.6006	432.8224	3900.1028	124.6005	-0.0000	-0.0013	0.0001
2382400.5	483.6476	3535.227	156.8733	483.6479	3535.2043	156.8737	-0.0003	-0.0018	-0.0004
2382800.5	-74.4947	3056.1399	178.0283	574.4481	3056.1410	178.0289	-0.0003	-0.0010	-0.0006
2383200.5	-92.2213	2520.6884	188.4597	692.2215	2520.6896	188.4598	-0.0002	-0.0012	-0.0000
2383600.5	-23.5973	1982.6383	189.6407	824.9572	1982.6385	189.6409	0.0001	-0.0003	-0.0002
2384000.5	962.9157	1482.8171	183.2695	962.9156	1482.8170	183.2695	0.0001	0.0001	0.0000
2384400.5	1109.7693	1047.0199	170.7698	1109.7686	1047.0704	170.7705	0.0006	-0.0006	-0.0008
2384800.5	1268.0262	686.1640	153.0683	1268.0303	686.1642	153.0683	-0.0004	-0.0002	-0.0003
2385200.5	1440.2171	395.1978	130.4710	1440.2167	395.1984	130.4712	0.0004	-0.0006	-0.0002
2385600.5	1.08.2703	152.5309	102.6978	1608.2713	152.5317	102.6976	-0.0010	-0.0007	0.0001
2386000.5	1723.2212	-76.9175	69.4399	1723.2221	-76.9170	69.4398	-0.0009	-0.0003	0.0003
2386400.5	182.3824	-309.2460	32.1256	1682.3826	-309.2456	32.1258	-0.0002	-0.0004	-0.0001
2386800.5	1400.7391	-645.4481	-5.7352	1400.7397	-645.4488	-5.7362	-0.0006	0.0007	0.0010
2387200.5	162.4008	-448.1944	-42.8749	962.4010	-448.1932	-42.8749	-0.0002	-0.0012	-0.0001
2387600.5	-96.8252	-266.820	-78.1534	506.8255	-266.8298	-78.1543	-0.0004	0.0007	0.0009
2388000.5	118.8753	19.2129	-108.5218	118.8749	19.2130	-108.5225	0.0004	-0.0001	0.0007
2388400.5	-164.3030	343.6671	-130.5582	-164.0297	343.6684	-130.5584	-0.0003	-0.0014	0.0001
2388800.5	-34.4346	644.1353	-141.6660	-334.4344	644.1355	-141.6659	-0.0001	-0.0002	-0.0001
2389200.5	-401.4805	868.4903	-140.6626	-401.4798	868.4903	-140.6626	-0.0006	0.0003	-0.0000
2389600.5	-379.0274	980.4402	-128.0651	-379.8267	980.4410	-128.0647	-0.0007	-0.0008	-0.0003
2390000.5	-781.0149	964.9458	-106.1239	-281.0143	964.9460	-106.1247	-0.0007	-0.0001	0.0008
2390400.5	-114.1370	832.1.30	-78.4481	-114.1377	832.1027	-78.4485	0.0007	0.0003	0.0004
2390800.5	115.0392	614.9422	-48.9788	115.6392	614.9421	-48.9795	-0.0000	0.0001	0.0007
2391200.5	.95.2628	356.3479	-20.6354	395.2825	356.3485	-20.6346	0.0004	-0.0006	-0.0008
2391600.5	707.8704	90.0736	5.4519	707.8711	90.0734	5.4528	-0.0007	0.0003	-0.0009
2392000.5	1.37.0119	-171.0.96	29.5459	1037.8124	-171.0702	29.5458	-0.0005	0.0000	0.0001
2392400.5	1.72.4804	-435.0187	52.3506	1372.4809	-435.0186	52.3505	-0.0005	-0.0001	0.0000
2392800.5	1.96.4427	-724.0157	74.2179	1696.4427	-724.0161	74.2177	0.0000	0.0005	0.0002
2393200.5	1.979.9829	-1066.5245	94.6371	1979.9826	-1066.5250	94.6362	0.0003	0.0006	0.0009
2393600.5	2163.2807	-1482.3928	111.9378	2163.2864	-1482.3929	111.9384	0.0003	0.0001	-0.0006
2394000.5	2158.1442	-1942.1102	123.9453	2158.1443	-1942.1109	123.9448	-0.0001	0.0007	0.0005
2394400.5	1.35.1116	-2349.3725	129.9416	1935.1113	-2349.3721	129.9418	0.0003	-0.0004	-0.0001
2394800.5	1.67.073	-2655.7199	129.2725	1567.0783	-2655.7206	129.2719	-0.0000	0.0006	0.0005
2395200.5	1.132.1822	-2887.0922	120.1142	1132.1828	-2887.0926	120.1142	-0.0007	0.0005	0.0000
2395600.5	.82.6838	-3074.5170	191.6243	682.6843	-3074.5180	191.6240	-0.0005	0.0010	0.0003
2396000.5	251.6861	-3225.0674	75.0329	251.6891	-3225.0672	75.0334	-0.0010	-0.0002	-0.0004
2396400.5	-143.2435	-3325.2784	43.1919	-143.2439	-3325.2786	43.1925	0.0005	0.0002	-0.0006
2396800.5	-494.1282	-3353.2453	9.6042	-494.1280	-3353.2658	9.6040	-0.0002	0.0005	0.0001
2397200.5	-794.5354	-3287.4712	-22.3390	-794.5351	-3287.7136	-22.3389	-0.0003	0.0004	-0.0001
2397600.5	-1.33.3493	-3112.3521	-49.8254	-1033.3490	-3112.3525	-49.8256	-0.0003	0.0005	0.0002
2398000.5	-1192.1652	-2818.0374	-70.9727	-1192.1650	-2818.0381	-70.9721	-0.0002	0.0007	-0.0006
2398400.5	-1246.9424	-2404.3783	-85.1072	-1246.9421	-2404.3087	-85.1070	-0.0003	0.0005	-0.0002
2398800.5	-1174.3933	-1882.0862	-92.8824	-1174.3931	-1882.0874	-92.8819	-0.0001	0.0012	-0.0005
2399200.5	-761.0888	-1279.0503	-95.9062	-961.0988	-1279.0517	-95.9063	0.0009	0.0014	0.0001
2399600.5	-16.5772	-646.5562	-95.7347	-616.5767	-646.5568	-95.7347	-0.0005	0.0006	-0.0000
2400000.5	-167.5569	-61.2179	-92.7358	-167.5563	-61.2178	-92.7363	-0.0004	-0.0001	0.0005
2400400.5	282.4645	387.1324	-85.9117	328.4645	387.1318	-85.9121	0.0000	0.0006	0.0004
2400800.5	770.0312	627.0654	-74.2385	770.0320	627.0651	-74.2389	-0.0008	0.0002	0.0004
2401200.5	1.07.5133	676.0.12	-59.2493	1007.6138	676.0016	-59.2492	-0.0005	-0.0004	-0.0001
2401600.5	172.4414	686.3356	-44.1265	972.4413	686.3359	-44.1267	0.0002	-0.0002	0.0001
2402000.5	176.1079	778.8254	-28.2920	768.1074	778.8241	-28.2914	0.0005	0.0013	-0.0006
2402400.5	11.11.1311	948.4231	-9.9966	511.1316	948.4223	-9.9966	-0.0005	0.0008	-0.0000
2402800.5	162.3263	1136.4928	10.4370	262.0259	1136.4930	10.4366	0.0004	-0.0002	0.0004
2403200.5	48.8128	1281.4156	30.8135	48.8126	1281.4156	30.8138	0.0003	-0.0000	-0.0004
2403600.5	-111.8435	1337.4680	48.0799	-111.8439	1337.4678	48.0797	0.0004	0.0001	0.0002
2404000.5	-108.3624	1294.9097	59.6658	-208.3631	1294.9091	59.6655	0.0007	0.0006	0.0003
2404400.5	-37.3461	1177.6506	64.6573	-237.3466	1177.6501	64.6572	0.0005	0.0004	0.0001
2404800.5	-204.6840	1027.2754	63.9553	-204.6849	1027.2755	63.9556	0.0009	0.0004	-0.0003
2405200.5	-119.7451	883.7620	59.4967	-119.7453	883.7610	59.4966	0.0001	0.0010	0.0001
2405600.5	11.6350	774.9136	53.3379	11.6343	774.9119	53.3383	0.0007	0.0017	-0.0004
2406000.5	189.7244	713.7306	47.0911	189.7241	713.7299	47.0913	0.0003	0.0007	-0.0002
2406400.5	418.3832	698.6533	41.6412	418.3835	698.6525	41.6416	-0.0003	0.0008	-0.0004
2406800.5	698.4670	713.6652	37.0023	698.4665	713.6652	37.0019	0.0004	-0.0000	0.0004
2407200.5	17.3660	727.6157	32.2784	1017.3591	727.6151	32.2780	0.0009	0.0006	0.0004
2407600.5	1.33.4368	695.0.90	26.0377	1333.4365	695.0790	26.0369	0.0003	0.0000	0.0008
2408000.5	1.53.9266	573.6726	17.7584	1553.9255	573.6719	17.7587	0.0011	0.0006	-0.0003
2408400.5	1.51.1332	395.8114	9.4797	1551.1321	395.8309	9.4798	0.0011	0.0004	-0.0001
2408800.5	1.26.8747	283.4135	2.0474	1326.8742	283.4131	2.0464	0.0004	0.0005	0.0010
2409200.5	1.16.6063	279.5608	-5.8031	1016.6058	279.5595	-5.8029	0.0005	0.0012	-0.0001
2409600.5	172.4628	346.5658	-13.4418	722.4622	346.5651	-13.4419	0.0006	0.0007	0.0001
2410000.5	490.8188	430.1568	-19.0950	490.8179	430.1555	-19.0950	0.0010	0.0014	0.0000
2410400.5	333.3873	479.3986	-21.1741	333.0870	479.3976	-21.1745	0.0003	0.0010	0.0004
2410800.5	242.206	453.4950	-18.8969	242.0206	453.4944	-18.8968	-0.0001	0.0007	-0.0000
2411200.5	204.3191	326.71.00	-12.5738	204.3188	326.6992	-12.5742	0.0003	0.0008	0.0004
2411600.5	209.9642	92.9923	-3.7254	209.9644	92.9920	-3.7263	-0.0002	0.0003	0.0008
2412000.5	156.0500	-229.9538	5.1297	256.0498	-229.9546	5.1299	0.0002	0.0008	-0.0002
2412400.5	343.2112	-601.3712	11.2681	343.2110	-601.3716	11.2681	0.0003	0.0004	0.0001
2412800.5	468.6934	-968.6465	12.9821	468.6937</					

Table C-2 (Continued)

Julian Day	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	$\alpha_T \cdot 10^6$	$\beta_T \cdot 10^6$	$\gamma_T \cdot 10^6$	$(\alpha - \alpha_T) \cdot 10^6$	$(\beta - \beta_T) \cdot 10^6$	$(\gamma - \gamma_T) \cdot 10^6$
2414800.5	1.22.460	-1964.3764	-7.3463	1422.4603	-1964.3764	-7.3477	-0.0003	0.0001	0.0014
2415200.5	1.84.0223	-2116.0311	-7.5232	1594.0220	-2116.0814	-7.5225	0.0002	0.0003	-0.0007
2415600.5	1.19.2873	-2305.7776	-6.9815	1619.2878	-2305.7785	-6.9815	-0.0005	0.0010	0.0000
2416000.5	1.62.3519	-2470.3571	-6.4083	1462.3517	-2470.3567	-6.4081	0.0001	-0.0004	-0.0002
2416400.5	1.156.0944	-2544.4682	-5.9111	1156.0956	-2544.4689	-5.9113	-0.0012	0.0008	0.0002
2416800.5	785.2510	-2547.4405	-7.0633	785.2511	-2547.4908	-7.0641	-0.0002	0.0003	0.0009
2417200.5	409.2064	-2525.2772	-11.2126	409.2071	-2525.2775	-11.2118	-0.0006	0.0003	-0.0008
2417600.5	63.6247	-2503.8337	-17.9597	63.6249	-2503.8843	-17.9600	-0.0001	0.0006	0.0003
2418000.5	-34.5908	-2485.3638	-25.4498	-234.5917	-2485.3652	-25.4500	0.0009	0.0014	0.0002
2418400.5	-81.7253	-2457.7492	-31.3284	-481.7252	-2457.7704	-31.3288	-0.0000	0.0012	0.0004
2418800.5	-79.0092	-2404.0216	-33.5585	-679.0092	-2404.0213	-33.5577	0.0000	-0.0003	-0.0008
2419200.5	-25.3366	-2306.5106	-30.8775	-825.0358	-2306.5306	-30.8782	-0.0008	0.0001	0.0007
2419600.5	-911.1739	-2149.4475	-23.0781	-911.1744	-2149.4881	-23.0780	0.0006	0.0006	-0.0001
2420000.5	-21.1066	-1920.7335	-11.1604	-921.0998	-1920.7637	-11.1605	-0.0008	0.0002	0.0001
2420400.5	-35.3336	-1614.8438	2.6560	-835.3368	-1614.8430	2.6565	0.0002	-0.0009	-0.0005
2420800.5	-40.3444	-1238.3703	15.3705	-640.3447	-1238.3708	15.3711	0.0004	0.0005	-0.0006
2421200.5	-337.6787	-818.4194	24.2603	-337.6791	-818.4189	24.2602	0.0004	-0.0004	0.0002
2421600.5	52.3779	-408.0695	28.1482	52.3775	-408.0707	28.1485	0.0004	0.0012	-0.0003
2422000.5	489.2602	-80.5336	27.8322	489.2603	-80.5642	27.8318	-0.0001	0.0006	0.0004
2422400.5	400.5164	92.0415	25.0653	900.5159	92.0406	25.0660	0.0005	0.0009	-0.0007
2422800.5	1162.6700	86.2205	20.2881	1162.6700	86.2210	20.2882	0.0000	-0.0004	-0.0001
2423200.5	1155.8951	3.5267	11.5935	1155.8952	3.5267	11.5930	-0.0001	0.0000	0.0005
2423600.5	118.1019	4.1c71	-0.7511	918.1018	4.1667	-0.7511	0.0001	0.0004	0.0000
2424000.5	-77.5321	134.3104	-12.8626	577.5317	134.3099	-12.8627	0.0004	0.0006	0.0001
2424400.5	19.6870	351.5923	-22.8460	219.6865	351.5917	-22.8456	0.0005	0.0005	-0.0004
2424800.5	-112.4586	591.9470	-31.0143	-112.4577	591.9461	-31.0146	-0.0009	0.0009	0.0004
2425200.5	-93.4763	801.4440	-38.9803	-393.4759	801.4485	-38.9809	-0.0004	-0.0005	0.0006
2425600.5	-694.7682	955.2972	-48.2999	-604.7683	955.2970	-48.2997	0.0001	0.0002	-0.0002
2426000.5	-735.9395	1062.8164	-59.2128	-735.9395	1062.8068	-59.2121	-0.0000	-0.0003	-0.0006
2426400.5	-787.0794	1154.8346	-70.3024	-787.0976	1154.8349	-70.3029	0.0003	-0.0003	0.0005
2426800.5	-764.4059	1264.1488	-79.1915	-764.4060	1264.1491	-79.1917	0.0001	-0.0003	0.0001
2427200.5	-72.6966	1412.9498	-83.4885	-672.6966	1412.9603	-83.4883	-0.0000	-0.0005	-0.0002
2427600.5	-111.5107	1609.0270	-81.4457	-511.5109	1609.0277	-81.4451	0.0002	-0.0008	-0.0005
2428000.5	-276.2597	1845.6127	-72.2906	-276.2606	1845.6332	-72.2911	0.0009	-0.0035	0.0005
2428400.5	36.3192	2101.9235	-56.4142	36.3184	2101.9239	-56.4146	0.0008	-0.0004	0.0004
2428800.5	-18.9152	2342.4216	-35.4455	418.9151	2342.4209	-35.4461	0.0000	0.0007	0.0006
2429200.5	40.1256	2516.6237	-12.0239	846.1249	2516.6237	-12.0233	0.0006	-0.0001	-0.0006
2429600.5	1.24.4564	2566.8399	11.3358	1224.4588	2566.8599	11.3359	-0.0004	0.0001	-0.0001
2430000.5	-1.38.1629	2477.7135	34.3833	1438.1633	2477.7128	34.3830	-0.0004	0.0007	0.0004
2430400.5	1.98.7238	2353.3336	57.5033	1398.7245	2353.3335	57.5036	-0.0006	0.0002	-0.0003
2430800.5	1207.5550	2291.9381	77.6798	1207.5557	2291.9384	77.6794	-0.0007	-0.0003	0.0004
2431200.5	789.8428	2283.5394	92.8370	989.8421	2283.5390	92.8370	0.0006	0.0004	0.0000
2431600.5	-98.7516	2279.6758	102.9638	808.7508	2279.6753	102.9642	0.0009	0.0005	-0.0004
2432000.5	-83.3779	2230.941	108.6708	683.3771	2230.9041	108.6715	0.0007	-0.0006	-0.0007
2432400.5	-109.0837	2097.1033	110.4564	609.0835	2097.1839	110.4558	0.0001	-0.0006	0.0006
2432800.5	-72.2420	1853.3232	108.3591	572.2416	1853.2326	108.3588	0.0004	-0.0003	0.0003
2433200.5	-61.1380	1493.1716	101.7603	561.1390	1493.1924	101.7604	-0.0010	-0.0008	-0.0001
2433600.5	71.6508	1935.0727	89.4093	571.6515	1935.0725	89.4097	-0.0006	0.0002	-0.0004
2434000.5	-95.3437	522.0177	69.9777	605.3432	522.0181	69.9779	0.0004	-0.0004	-0.0001
2434400.5	-62.3857	13.9790	43.1274	662.3859	13.9784	43.1270	-0.0002	0.0006	0.0004
2434800.5	-138.1445	-431.2438	10.3372	738.1444	-431.2441	10.3368	0.0001	0.0004	0.0004
2435200.5	-27.3435	-776.036	-25.3143	821.3444	-776.0836	-25.3144	-0.0010	0.0000	0.0000
2435600.5	-28.4700	-1009.6402	-60.1465	928.4698	-1009.6400	-60.1469	0.0002	-0.0001	0.0004
2436000.5	1.41.7143	-1144.3657	-90.8444	1041.7138	-1144.3656	-90.8437	0.0004	-0.0001	-0.0007
2436400.5	1160.8239	-1209.6726	-115.0512	1160.8238	-1209.6725	-115.0517	0.0001	-0.0001	0.0005
2436800.5	1.60.2189	-1244.2329	-131.7361	1260.2194	-1244.2334	-131.7359	-0.0005	0.0005	-0.0002
2437200.5	1.78.7929	-1277.7422	-141.2444	1278.7940	-1277.7816	-141.2447	-0.0011	-0.0006	0.0004
2437600.5	1.135.7045	-1284.0430	-144.1934	1135.7050	-1284.0426	-144.1935	-0.0005	-0.0003	0.0001
2438000.5	-34.1649	-1190.7160	-140.4794	834.1607	-1190.7360	-140.4795	0.0003	0.0000	0.0001
2438400.5	461.2027	-997.5424	-131.4072	461.2030	-997.5422	-131.4069	-0.0003	-0.0002	-0.0003
2438800.5	90.3509	-759.0834	-118.8917	39.3510	-759.0826	-118.8924	-0.0002	-0.0008	0.0007
2439200.5	-133.7739	-521.6197	-103.5362	-233.7735	-521.6193	-103.5362	-0.0004	-0.0004	-0.0000
2439600.5	-689.3459	-307.7724	-84.6388	-489.3457	-307.7612	-84.6387	-0.0002	-0.0011	-0.0001
2440000.5	-71.9415	-122.9334	-61.1505	-671.9408	-122.9297	-61.1511	-0.0007	-0.0007	0.0006
2440400.5	-786.5581	36.1923	-32.5222	-786.5582	36.1931	-32.5222	0.0001	-0.0007	0.0000
2440800.5	-539.3629	178.0457	0.8439	-839.3624	178.0469	0.8443	-0.0005	-0.0012	-0.0004
2441200.5	-31.7133	313.9012	37.4066	-831.7137	313.9017	37.4074	0.0004	-0.0005	-0.0007
2441600.5	-157.7653	457.0435	74.4543	-757.7660	457.0427	74.4551	0.0007	0.0008	-0.0008
2442000.5	-606.7348	620.5378	108.2417	-606.7345	620.5876	108.2421	-0.0003	0.0001	-0.0003
2442400.5	-70.1132	812.6167	134.5037	-370.1127	812.6168	134.5029	-0.0005	-0.0001	0.0007
2442800.5	-50.4175	1027.4263	149.5492	-50.4177	1027.4259	149.5494	0.0002	0.0004	-0.0002
2443200.5	334.4110	1236.6661	151.6267	334.4107	1236.6667	151.6263	0.0003	-0.0007	0.0003
2443600.5	753.5700	1389.3326	141.5220	753.5707	1389.3330	141.5223	-0.0007	-0.0004	-0.0002
2444000.5	1.153.1226	1425.4609	121.7791	1153.1233	1425.4612	121.7793	-0.0007	-0.0003	-0.0002
2444400.5	1.41.2549	1306.4660	94.7289	1441.2553	1306.4662	94.7286	-0.0004	-0.0002	0.0003
2444800.5	1489.7418	1080.3034	60.4434	1489.7417	1080.3033	60.4439	0.0000	0.0002	-0.0004
2445200.5	1.59.8959	907.227	19.4523	1259.8964	907.2525	19.4525	-0.0005	0.0002	-0.0002
2445600.5	-61.4811	895.5580	-23.5613	861.4808	895.5583	-23.5612	0.0003	-0.0004	-0.0001
2446000.5	405.4187	1035.1676	-64.2833	405.4191	1035.1677	-64.2834	-0.0004	-0.0001	0.0001
2446400.5	-45.4221	1268.9288	-101.0632	-45.4221	1268.9292	-101.0625	0.0000	-0.0003	-0.0007
2446800.5	-453.4141	1537.6356	-134.0353	-453.4149	1537.6356	-134.0355	0.0008	0.0000	0.0002
2447200.5	-790.7485	1803.7763	-163.7314	-790.7495	1803.7072	-163.7324	0.0010	-0.0009	0.0009
2447600.5	-1.38.5042	2061.4566	-189.7266	-1038.5043	2061.4569	-189.7263	0.0002	-0.0003	-0.0004
2448000.5	-1189.7908	2329.0823	-210.1123	-1189.7913	2329.0822	-210.1124	0.0005	0.0000	0.0001
2448400.5	-1.46.8349	2630.4797	-222.0953	-1246.8345	2630.4796	-222.0946	-0.0004	0.0001	-0.0006
2448800.5	-1.213.3770	2981.4217	-222.9820	-1213.3769	2981.4222	-222.9820	-0.0001	-0.0006	0.0001
2449200.5	-1.89.2225	3384.3262	-210.9335	-1089.2228	3384.3267	-210.9332	0.0003	-0.0005	-0.0003
2449600.5	-669.8365	3827.7109	-185.3595	-869.8373	3827.7120	-185.3595	0.0008	-0.0011	0.0000
2450000.5	-550.5557	4286.1552	-147.1489						

Table C-3
Residuals for Uranus.

Julian Day	$\alpha \cdot 10^6$	$\delta \cdot 10^6$	$\gamma \cdot 10^6$	$\alpha_r \cdot 10^6$	$\beta_r \cdot 10^6$	$\gamma_r \cdot 10^6$	$(\alpha - \alpha_r) \cdot 10^6$	$(\beta - \beta_r) \cdot 10^6$	$(\gamma - \gamma_r) \cdot 10^6$
2378400.5	933.1645	345.7562	42.6142	933.1643	345.7557	42.6152	0.0002	0.0005	-0.0010
2378800.5	1138.2101	304.3276	42.0395	1138.2097	304.3279	42.0392	0.0005	-0.0003	0.0004
2379200.5	1309.7664	306.0674	41.3504	1309.7058	306.0674	41.3506	0.0006	-0.0000	-0.0002
2379600.5	1424.1168	331.9903	40.0087	1424.0168	331.9904	40.0088	0.0000	-0.0001	-0.0001
2380000.5	1468.0792	361.4913	37.6573	1468.0792	361.4911	37.6576	0.0000	0.0002	-0.0002
2380400.5	1440.5844	374.9632	34.1601	1440.5841	374.9032	34.1600	0.0003	-0.0001	0.0001
2380600.5	1352.2837	352.8895	29.6046	1352.2838	352.8893	29.6046	-0.0001	0.0006	-0.0001
2381200.5	1225.2731	278.3664	24.3262	1225.2735	278.3662	24.3256	-0.0003	0.0003	0.0005
2381600.5	1091.9193	142.8506	18.8802	1091.9191	142.8509	18.8800	0.0001	-0.0003	0.0001
2382000.5	988.5210	-45.5605	13.9102	988.5215	-45.5607	13.9102	-0.0005	0.0003	0.0000
2382400.5	941.5869	-259.7534	9.8737	941.5874	-259.7540	9.8735	-0.0005	0.0005	0.0002
2382800.5	955.9047	-463.0436	6.092	955.905	-463.0437	6.0888	-0.0003	0.0001	0.0005
2383200.5	1315.5994	-624.4116	4.3573	1015.5093	-624.4112	4.3577	0.0001	-0.0004	-0.0004
2383600.5	1094.3726	-727.2626	1.9908	1094.3724	-727.2627	1.9909	0.0002	0.0002	-0.0000
2384000.5	1165.8740	-769.2490	-0.7715	1165.8742	-769.2487	-0.7713	-0.0003	-0.0003	-0.0002
2384400.5	1206.7793	-758.6137	-4.2374	1206.7787	-758.6141	-4.2371	0.0006	0.0003	-0.0003
2384800.5	1198.3317	-711.1602	-8.4942	1198.3317	-711.1596	-8.4949	-0.0001	-0.0006	0.0007
2385200.5	1128.7434	-648.7388	-13.3945	1128.7438	-648.7388	-13.3947	-0.0004	-0.0000	0.0002
2385600.5	998.5850	-599.0518	-18.5365	998.5848	-599.0521	-18.5366	0.0002	0.0003	0.0001
2386000.5	825.957	-592.7118	-23.2867	825.956	-592.7119	-23.2870	0.0001	0.0002	0.0003
2386400.5	641.735	-652.5639	-26.9390	641.734	-652.5636	-26.9393	-0.0006	-0.0003	0.0002
2386800.5	485.053	-779.7928	-29.0292	485.0496	-779.7927	-29.0292	0.0005	-0.0002	-0.0001
2387200.5	365.5599	-952.3525	-29.5632	385.5599	-952.3524	-29.5631	-0.0001	-0.0001	-0.0000
2387600.5	351.8763	-1136.2421	-28.9605	351.8760	-1136.2420	-28.9604	0.0002	-0.0000	-0.0001
23880.5	376.4483	-1299.1469	-27.8145	376.4481	-1299.1470	-27.8148	0.0002	0.0000	0.0003
2388400.5	442.7443	-1417.2584	-26.6873	442.0739	-1417.2583	-26.6874	0.0003	-0.0001	0.0001
2388800.5	527.535	-1476.5683	-25.9942	527.0534	-1476.5811	-25.9946	0.0001	0.0007	0.0004
2389200.5	607.3133	-1472.9811	-25.9502	607.3137	-1472.9808	-25.9503	-0.0004	-0.0003	0.0001
2389600.5	657.4993	-1413.2910	-26.5229	657.6988	-1413.2913	-26.5230	0.0004	0.0004	0.0001
23900.5	655.7938	-1317.3782	-27.4118	655.7943	-1317.0784	-27.4115	-0.0005	0.0003	-0.0003
2390400.5	590.8511	-1216.2828	-28.0930	590.8517	-1216.2834	-28.0930	-0.0005	0.0007	0.0000
2390800.5	473.1052	-1148.3705	-28.0071	473.1053	-1148.3705	-28.0068	-0.0001	-0.0000	-0.0003
2391200.5	332.4955	-1140.9181	-26.8520	332.4957	-1140.9180	-26.8518	-0.0001	-0.0000	-0.0002
2391600.5	205.2849	-1198.4209	-24.7685	205.2852	-1198.4204	-24.7688	-0.0003	-0.0004	0.0002
2392000.5	119.5402	-1303.6010	-22.2389	119.5401	-1303.6012	-22.2388	0.0002	0.0001	-0.0001
2392400.5	89.2725	-1428.2455	-19.8411	89.2728	-1428.2458	-19.8411	-0.0003	0.0003	-0.0000
2392800.5	116.0796	-1542.6390	-18.0654	116.0800	-1542.6386	-18.0651	-0.0004	-0.0005	-0.0002
2393200.5	192.3624	-1620.2483	-17.2181	192.3623	-1620.2485	-17.2180	0.0001	0.0002	-0.0001
2393600.5	302.8904	-1639.9978	-17.3773	302.8906	-1639.9977	-17.3775	-0.0001	-0.0001	0.0001
2394000.5	424.8233	-1589.1917	-18.3562	424.8226	-1589.1918	-18.3560	0.0004	0.0002	-0.0002
2394400.5	528.6330	-1468.6497	-19.6879	528.6334	-1468.6499	-19.6876	-0.0004	0.0002	-0.0003
2394800.5	583.9874	-1297.8350	-20.7093	583.9875	-1297.8343	-20.7090	-0.0001	-0.0007	-0.0003
2395200.5	572.4142	-1113.2074	-20.7930	572.4135	-1113.2076	-20.7930	0.0003	0.0002	0.0000
2395600.5	498.6369	-955.8557	-19.6364	498.6373	-955.8555	-19.6362	-0.0003	-0.0002	-0.0002
2396000.5	387.4192	-855.1215	-17.3865	387.4193	-855.1213	-17.3863	0.0002	-0.0003	-0.0002
2396400.5	269.7039	-820.0182	-14.5169	269.7040	-820.0183	-14.5170	-0.0002	0.0001	0.0001
2396800.5	171.7327	-841.9054	-11.5943	171.7325	-841.9055	-11.5943	0.0002	0.0001	-0.0001
2397200.5	111.4188	-901.2923	-9.1017	111.4191	-901.2919	-9.1017	-0.0003	-0.0003	0.0001
2397600.5	98.7695	-972.9201	-7.3433	98.7695	-972.9207	-7.3432	0.0000	0.0005	-0.0001
2398000.5	135.9527	-1028.7995	-6.3827	135.9526	-1028.8000	-6.3825	0.0001	0.0006	-0.0002
2398400.5	216.2265	-1041.4017	-6.0055	216.2270	-1041.4014	-6.0057	-0.0005	-0.0003	0.0002
2398800.5	320.3638	-989.5284	-5.7312	320.3636	-989.5287	-5.7309	0.0002	0.0003	-0.0003
2399200.5	417.0470	-867.4908	-4.9276	417.0475	-867.4909	-4.9269	-0.0005	0.0001	-0.0007
2399600.5	471.4676	-692.1633	-3.0799	471.4677	-692.1635	-3.0796	-0.0002	0.0002	-0.0003
2400000.5	460.8647	-498.2754	-0.0714	460.8646	-498.2757	-0.0716	0.0005	0.0003	0.0002
2400400.5	385.1720	-323.1016	3.7683	385.1723	-323.1016	3.7682	-0.0003	-0.0000	0.0001
2400800.5	263.3317	-194.1108	7.8486	263.3311	-194.1108	7.8491	0.0006	-0.0000	-0.0005
2401200.5	121.8642	-125.7158	11.5429	121.8654	-125.0162	11.5430	-0.0012	0.0004	-0.0001
2401600.5	-12.8249	-117.0516	14.3417	-12.8253	-117.0508	14.3416	0.0004	-0.0009	0.0001
2402000.5	-117.1509	-161.4674	15.9383	-117.1510	-161.4672	15.9387	0.0000	-0.0001	-0.0004
2402400.5	-171.4101	-240.9138	16.2763	-171.4100	-240.9137	16.2769	-0.0001	-0.0001	-0.0006
2402800.5	-162.2934	-330.0018	15.5834	-162.2931	-330.0021	15.5835	-0.0004	0.0003	-0.0001
2403200.5	-85.2971	-397.7430	14.3662	-88.2977	-397.7427	14.3659	0.0006	-0.0003	-0.0003
2403600.5	34.2106	-415.4419	13.2886	34.2103	-415.4419	13.2883	0.0003	0.0001	0.0003
2404000.5	171.6204	-369.3591	12.8917	171.6203	-369.3586	12.8914	0.0002	-0.0006	0.0003
2404400.5	285.1608	-268.4665	13.3151	285.1603	-268.4665	13.3147	0.0005	0.0000	0.0003
2404800.5	346.4411	-138.4648	14.2693	346.4410	-138.4646	14.2689	0.0000	-0.0002	0.0004
2405200.5	345.3504	-8.1515	15.2562	345.3497	-8.1518	15.2560	0.0007	0.0003	0.0002
2405600.5	288.9667	98.7462	15.7932	288.9672	98.7460	15.7931	-0.0006	0.0002	0.0001
2406000.5	194.9836	164.6274	15.5240	194.9837	164.6283	15.5239	-0.0001	-0.0008	0.0002
2406400.5	86.0357	178.2612	14.2811	86.0358	178.2615	14.2814	-0.0001	-0.0003	-0.0002
2406800.5	-12.2012	136.2019	12.1373	-12.2014	136.2021	12.1372	0.0002	-0.0002	0.0001
2407200.5	-72.3645	45.4763	9.4376	-72.3647	45.4761	9.4373	0.0002	0.0003	0.0003
2407600.5	-70.2940	-73.5776	6.7827	-70.2946	-73.5774	6.7828	0.0006	-0.0003	-0.0001
2408000.5	5.1090	-188.6367	4.8772	5.1087	-188.6366	4.8775	0.0003	-0.0001	-0.0003
2408400.5	142.9592	-265.5465	4.2264	142.9593	-265.5464	4.2268	-0.0002	-0.0001	-0.0004
2408800.5	312.8795	-283.0908	4.8822	312.8800	-283.0908	4.8822	-0.0005	0.0001	0.0001
2409200.5	478.6990	-240.1001	6.4579	478.6993	-240.0998	6.4577	-0.0003	-0.0002	0.0002
2409600.5	610.7222	-150.8452	8.3624	610.7218	-150.8460	8.3627	0.0004	0.0008	-0.0003
2410000.5	689.9767	-36.1637	10.0324	689.9768	-36.1639	10.0320	-0.0001	0.0001	0.0004
2410400.5	708.9042	82.5662	11.0426	708.9039	82.5662	11.0425	0.0002	-0.0000	0.0002
2410800.5	671.8719	184.2212	11.1449	671.8724	184.2215	11.1454	-0.0006	-0.0003	-0.0005
2411200.5	593.9791	247.4526	10.3160	593.9746	247.4525	10.3159	0.0005	0.0001	0.0001
2411600.5	499.7572	254.3063	8.8168	499.7572	254.3058	8.8168	-0.0000	0.0003	-0.0001
2412000.5	421.3295	197.8880	7.1851	421.3259	197.8882	7.1854	-0.0004	-0.0002	-0.0004
2412400.5	390.3445	89.9500	6.0594	390.3445	89.9495	6.0595	0.0001	0.0005	-0.0001
2412800.5	424.0942	-39.6060	5.8688	424.0935	-39.6054	5.8691	0.0007	-0.0006	-0.0002
2413200.5	516.7410	-154.9364	6.6190	516.7413	-154.9366	6.6194	-0.0003	0.0001	-0.0004
2413600.5	644.3083	-228.7229	7.9470	644.3085	-228.7222	7.9472	-0.0002	-0.0007</	

Table C-3 (Continued)

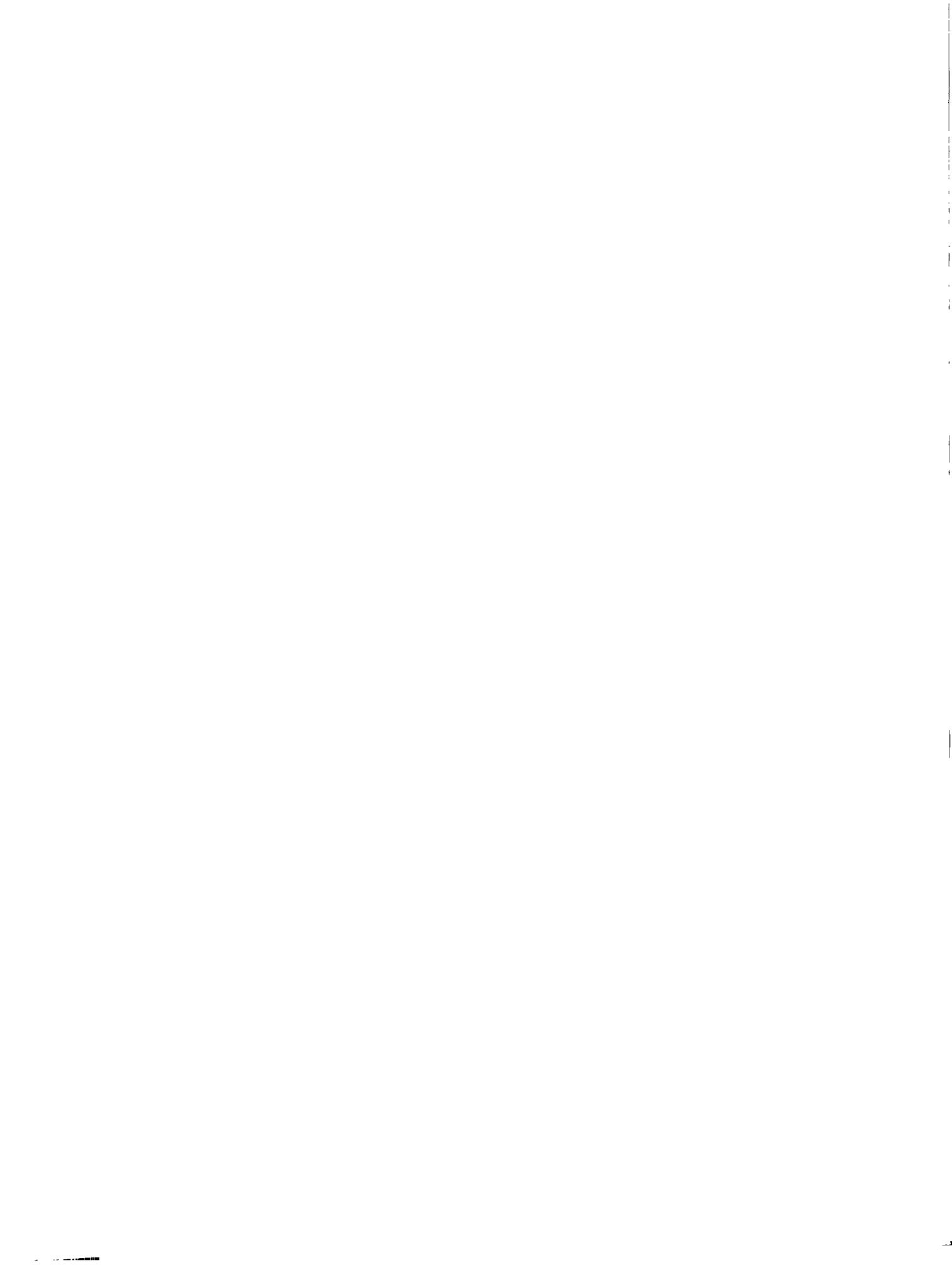
Julian Day	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	$\alpha_T \cdot 10^6$	$\beta_T \cdot 10^6$	$\gamma_T \cdot 10^6$	$(\alpha - \alpha_T) \cdot 10^6$	$(\beta - \beta_T) \cdot 10^6$	$(\gamma - \gamma_T) \cdot 10^6$
2415200.5	953.1628	-21.6277	9.7536	953.1629	-21.6268	9.7533	-0.0001	-0.0009	0.0003
2415600.5	887.8847	96.1409	8.1307	887.8840	96.1407	8.1308	0.0006	0.0002	-0.0001
2416000.5	761.2878	193.2866	5.9852	761.2883	193.2859	5.9852	-0.0004	0.0006	0.0000
2416400.5	596.6590	240.8741	3.9197	596.6591	240.8744	3.9195	0.0010	-0.0003	0.0002
2416800.5	430.4664	221.6535	2.5903	430.4664	221.6542	2.5904	0.0000	-0.0007	-0.0001
2417200.5	299.9925	141.2039	2.3855	299.9920	141.2043	2.3859	0.0005	-0.0005	-0.0004
2417600.5	228.7261	25.8932	3.2414	228.7264	25.8933	3.2412	-0.0003	-0.0001	0.0002
2418000.5	220.2414	-90.4616	4.7415	220.2418	-90.4618	4.7414	-0.0004	0.0002	0.0001
2418400.5	262.7696	-178.8055	6.3586	262.7693	-178.8048	6.3585	0.0003	-0.0007	0.0001
2418800.5	337.5519	-219.8213	7.6373	337.5519	-219.8213	7.6367	0.0000	0.0003	0.0006
2419200.5	421.3936	-204.3599	8.2833	421.3935	-204.3604	8.2831	-0.0003	0.0005	0.0002
2419600.5	490.4526	-133.2988	8.2116	490.4532	-133.2993	8.2119	-0.0006	0.0006	-0.0003
2420000.5	520.8743	-18.5019	7.5739	520.8735	-18.5012	7.5741	0.0008	-0.0008	-0.0002
2420400.5	493.8955	116.0911	6.7636	493.8956	116.0905	6.7642	-0.0001	0.0006	-0.0006
2420800.5	405.3230	236.1135	6.3481	405.3226	236.1145	6.3483	0.0004	-0.0011	-0.0001
2421200.5	272.4620	305.6370	6.8561	272.4623	305.6371	6.8558	-0.0003	-0.0001	0.0003
2421600.5	129.1776	333.1733	8.4861	129.1778	333.1732	8.4862	-0.0002	0.0002	-0.0000
2422000.5	11.1339	231.7159	10.9785	11.1033	231.7162	10.9784	0.0006	-0.0003	0.0001
2422400.5	-57.2793	113.9433	13.7591	-57.280C	113.9435	13.7591	0.0007	-0.0003	-0.0000
2422800.5	-66.1826	-19.3738	16.1912	-66.1826	-19.3734	16.1909	0.0000	-0.0004	0.0003
2423200.5	-17.7612	-137.9720	17.7505	-17.7612	-137.9719	17.7497	-0.0000	-0.0001	0.0009
2423600.5	77.1934	-216.0826	18.1116	77.1935	-216.0827	18.1114	-0.0002	0.0001	0.0002
2424000.5	200.7245	-234.5695	17.1962	200.7244	-234.5694	17.1967	0.0000	-0.0001	-0.0005
2424400.5	327.9773	-184.0271	15.2042	327.9779	-184.0273	15.2043	-0.0005	0.0002	-0.0001
2424800.5	429.0279	-69.7696	12.6134	429.0275	-69.7698	12.6135	0.0004	0.0002	-0.0002
2425200.5	476.1925	84.3324	10.0809	476.1927	84.3327	10.0813	-0.0002	-0.0003	-0.0004
2425600.5	457.2422	239.8171	8.1945	457.2814	239.8170	8.1943	0.0008	0.0001	0.0002
2426000.5	384.2320	357.5822	7.1928	384.2320	357.5824	7.1926	0.0001	-0.0002	0.0002
2426400.5	286.2353	413.6849	6.8797	286.2356	413.6855	6.8796	-0.0003	-0.0006	0.0001
2426800.5	195.2502	405.2266	6.7832	195.2504	405.2265	6.7831	-0.0002	0.0001	0.0001
2427200.5	136.4795	345.3899	6.3930	136.4799	345.3897	6.3928	-0.0004	0.0002	0.0002
2427600.5	125.8907	256.0444	5.3179	125.8904	256.0442	5.3175	0.0003	0.0002	0.0003
2428000.5	170.7269	162.9194	3.3576	170.7276	162.9195	3.3577	-0.0007	-0.0000	-0.0001
2428400.5	269.4574	92.9585	0.5437	269.4572	92.9587	0.5440	0.0002	-0.0002	-0.0003
2428800.5	409.8968	71.2386	-2.8328	409.8970	71.2381	-2.8327	-0.0002	0.0005	-0.0001
2429200.5	567.0071	114.7662	-6.2317	567.0069	114.7662	-6.2320	0.0003	0.0001	0.0003
2429600.5	704.9621	223.4882	-8.9911	704.9619	223.4874	-8.9914	0.0002	0.0008	0.0003
2430000.5	788.1367	375.0701	-10.6056	788.1368	375.0702	-10.6052	-0.0002	-0.0001	-0.0003
2430400.5	757.2321	533.0551	-10.9795	757.2323	533.0549	-10.9795	0.0001	0.0003	-0.0001
2430800.5	737.3057	662.6881	-10.4310	737.3058	662.6884	-10.4313	-0.0000	-0.0003	0.0002
2431200.5	640.7497	741.2892	-9.4895	640.7497	741.2895	-9.4896	0.0000	-0.0003	0.0001
2431600.5	505.4701	760.0717	-8.6939	505.4702	760.0718	-8.6942	-0.0002	-0.0001	0.0003
2432000.5	388.1398	721.6746	-8.4747	388.1397	721.6744	-8.4751	0.0001	0.0002	0.0003
2432400.5	302.0404	637.6716	-9.0879	302.0408	637.6713	-9.0878	-0.0008	0.0003	-0.0001
2432800.5	265.6514	527.6069	-10.5631	265.6512	527.6068	-10.5630	0.0002	0.0001	-0.0002
2433200.5	289.8791	418.345C	-12.6612	289.8789	418.3454	-12.6613	0.0002	-0.0004	0.0001
2433600.5	371.8622	340.5424	-14.8802	371.8625	340.5423	-14.8801	-0.0003	0.0002	-0.000J
2434000.5	490.1894	319.1404	-16.5995	490.1889	319.1402	-16.5995	0.0005	-0.0002	-0.0000
2434400.5	608.5462	360.8621	-17.3779	608.546C	360.8621	-17.3783	0.0002	0.0000	0.0004
2434800.5	690.0621	449.8515	-17.2013	690.0618	449.8516	-17.2014	0.0003	-0.0001	0.0001
2435200.5	711.7135	557.2596	-16.4491	711.7132	557.2594	-16.4489	0.0003	0.0002	-0.0002
2435600.5	669.5860	654.7049	-15.6579	669.5861	654.7044	-15.6581	-0.0001	0.0005	0.0002
2436000.5	575.3541	720.0798	-15.3231	575.3547	720.0799	-15.3233	-0.0006	0.0000	-0.0000
2436400.5	448.9301	737.9065	-15.7966	448.9296	737.9070	-15.7964	0.0005	-0.0006	-0.0002
2436800.5	313.9153	699.7690	-17.2277	313.9151	699.7691	-17.2276	0.0002	-0.0000	-0.0002
2437200.5	196.3313	605.8421	-19.5137	196.3315	605.842J	-19.5134	-0.0002	0.0001	-0.0003
2437600.5	122.6854	467.5896	-22.2647	122.6857	467.5902	-22.2647	-0.0003	-0.0005	-0.0000
2438000.5	113.7752	309.7554	-24.8397	113.7753	309.7556	-24.8398	-0.0001	-0.0002	0.0001
2438400.5	174.4441	166.6939	-26.5284	174.4443	166.6942	-26.5284	0.0001	-0.0003	-0.0000
2438800.5	287.1774	70.1136	-26.8620	287.1776	70.1138	-26.8620	-0.0002	-0.0002	-0.000J
2439200.5	418.4331	35.4773	-25.8291	418.4336	35.4774	-25.8291	-0.0005	0.0000	0.0000
2439600.5	533.2490	58.5255	-23.8075	533.2486	58.5253	-23.8075	0.0004	0.0002	0.0000
2440000.5	605.5168	122.2054	-21.3235	605.5171	122.2055	-21.3229	-0.0003	-0.0002	-0.0006
2440400.5	620.7693	205.5955	-18.8420	620.7689	205.5955	-18.8421	0.0004	0.0000	0.0001
2440800.5	576.7113	288.5515	-16.6881	576.7116	288.5517	-16.6886	-0.0003	-0.0003	0.0005
2441200.5	482.8338	351.4410	-15.0208	482.8335	351.4407	-15.0205	0.0003	0.0003	-0.0003
2441600.5	358.3738	375.5102	-13.7805	358.3736	375.5101	-13.7806	0.0002	0.0001	0.0001
2442000.5	231.1255	347.5555	-12.6498	231.1254	347.5544	-12.6502	0.0000	0.0011	0.0003
2442400.5	134.5150	267.4694	-11.0940	134.5150	267.4696	-11.0940	0.0000	-0.0002	-0.0000
2442800.5	97.9299	153.8967	-8.5659	97.9302	153.8969	-8.5656	-0.0003	-0.0002	-0.0003
2443200.5	133.2454	40.2894	-4.8140	133.2453	40.2893	-4.8139	0.0001	0.0000	0.0000
2443600.5	229.4366	-39.1958	-0.0477	229.4368	-39.1955	-0.0476	-0.0002	-0.0003	-0.0001
2444000.5	360.2528	-62.7066	5.1807	360.2524	-62.7064	5.1810	0.0004	-0.0002	-0.0003
2444400.5	495.8241	-24.1543	10.2277	495.8248	-24.1549	10.2279	-0.0007	0.0006	-0.0002
2444800.5	609.4637	70.0206	14.5547	609.4635	70.0207	14.5549	0.0003	-0.0001	-0.0003
2445200.5	679.5465	205.0802	17.8293	679.5465	205.0799	17.8294	-0.0000	0.0002	-0.0001
2445600.5	690.7821	360.8169	19.9626	690.7825	360.8166	19.9625	-0.0004	0.0003	0.0001
2446000.5	638.0471	512.4504	21.1240	638.0465	512.4508	21.1238	0.0005	-0.0004	0.0002
2446400.5	530.7256	631.3499	21.7572	530.7257	631.3494	21.7576	-0.0001	0.0005	-0.0003
2446800.5	393.8094	691.7297	22.5242	393.8094	691.7297	22.5245	0.0001	0.0000	-0.0002
2447200.5	263.1028	683.5728	24.0598	263.1029	683.5724	24.0594	-0.0002	0.0004	0.0004
2447600.5	172.2628	620.9469	26.6426	172.2629	620.9472	26.6428	-0.0001	-0.0003	-0.0003
2448000.5	139.1553	536.0496	30.6663	139.155C	536.0496	30.6664	0.0003	-0.0000	-0.0001
2448400.5	162.5774	464.2485	33.7960	162.5773	464.2487	33.7956	0.0001	-0.0002	0.0004
2448800.5	228.2669	432.8834	37.2371	228.2669	432.8831	37.2371	-0.0001	0.0003	-0.0000
2449200.5	315.8718	457.8180	39.9236	315.8720	457.8178	39.9233	-0.0002	0.0002	0.0003
2449600.5	402.7390	543.9065	41.5933	402.7392	543.9064	41.5936	-0.0002	0.0002	-0.0003
2450000.5	465.3989	685.4667	42.2290	465.3985	685.4662	42.2290	0.0004	0.0005	-0.0000
2450400.5	481.6447	865.6502	42.0740	481.6444	865.6508	42.0738	0.0003	-0.0005	0.0002
2450800.5	436.5356	1056.0209	41.6260	436.5351	1056.0210	41.6261	0.0005	-0.0001	-0.0001
2451200.5</td									

Table C-4
Residuals for Neptune.

Julian Day	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	$\alpha_t \cdot 10^6$	$\beta_t \cdot 10^6$	$\gamma_t \cdot 10^6$	$(\alpha - \alpha_t) \cdot 10^6$	$(\beta - \beta_t) \cdot 10^6$	$(\gamma - \gamma_t) \cdot 10^6$
2378400.5	-639.5330	-3534.4673	-4.5743	-639.5331	-3534.4676	-4.5748	0.0001	0.0003	0.0004
2378200.5	-557.2915	-3528.8501	-4.4714	-557.2917	-3528.8501	-4.4715	0.0002	0.0001	0.0001
2379200.5	-455.5662	-3476.8828	-5.2179	-455.5662	-3476.8826	-5.2175	-0.0003	-0.0002	-0.0004
2379400.5	-357.6766	-3377.2208	-6.4644	-357.6766	-3377.2210	-6.4646	-0.0006	0.0002	0.0002
2380000.5	-283.3224	-3238.7007	-7.7793	-283.3229	-3238.7012	-7.7794	0.0005	0.0005	0.0001
2380400.5	-248.6431	-3077.2640	-8.7425	-248.6432	-3077.2646	-8.7428	0.0002	0.0006	0.0003
2380800.5	-58.6423	-2913.8036	-9.0267	-258.6425	-2913.8036	-9.0267	0.0002	0.0001	0.0001
2381200.5	-310.5693	-2771.6666	-8.4807	-310.5691	-2771.6666	-8.4808	-0.0001	-0.0001	0.0001
2381600.5	-388.6464	-2671.4142	-7.2281	-388.6469	-2671.4141	-7.2281	0.0001	-0.0000	-0.0000
2382000.5	-466.3667	-2622.1372	-5.6976	-466.3667	-2622.1373	-5.6976	-0.0000	0.0000	-0.0000
2382400.5	-515.7711	-2614.5253	-4.4815	-515.7714	-2614.5251	-4.4810	0.0003	-0.0001	-0.0005
2382800.5	-520.3743	-2623.8952	-4.4433	-520.3747	-2623.8954	-4.4348	0.0005	0.0002	0.0005
2383200.5	-481.1324	-2622.3341	-4.5147	-481.1322	-2622.3337	-4.5148	-0.0004	-0.0004	0.0001
2383600.5	-412.3211	-2589.4967	-5.6959	-412.3206	-2589.4972	-5.6959	-0.0006	0.0005	-0.0001
238400.5	-334.3205	-2516.2829	-7.1981	-334.0205	-2516.2826	-7.1977	-0.0000	-0.0003	-0.0004
2384400.5	-266.7401	-2403.8841	-8.5731	-266.7399	-2403.8842	-8.5727	-0.0001	0.0001	-0.0004
2384800.5	-229.4829	-2262.0672	-9.4075	-228.4824	-2262.0671	-9.4075	-0.0005	-0.0001	-0.0001
2385200.5	-232.2470	-2108.0441	-9.4032	-232.2471	-2108.0441	-9.3998	0.0002	-0.0000	-0.0003
2385600.5	-282.3063	-1965.0434	-8.4475	-282.3067	-1965.0432	-8.4471	-0.0002	-0.0001	-0.0004
238600.5	-369.7460	-1857.9774	-6.7369	-369.7462	-1857.9775	-6.7371	0.0002	0.0002	0.0001
2386400.5	-471.2446	-1804.2698	-4.7584	-471.2443	-1804.2693	-4.7584	-0.0003	-0.0007	-0.0000
2386800.5	-556.2649	-1803.9917	-3.1237	-556.2649	-1803.9915	-3.1235	-0.0000	-0.0002	-0.0002
2387200.5	-600.5594	-1838.8398	-2.2742	-600.5594	-1838.8399	-2.2740	0.0001	0.0002	-0.0002
2387600.5	-595.3225	-1881.7869	-2.3054	-595.3232	-1881.7865	-2.3053	0.0001	-0.0003	-0.0000
238800.5	-546.3139	-1908.1305	-3.0090	-546.3138	-1908.1308	-3.0088	-0.0001	0.0002	-0.0002
2388400.5	-467.7054	-1901.0621	-3.1119	-467.7048	-1901.0613	-4.0116	-0.0005	-0.0008	-0.0003
2388800.5	-379.3566	-1852.7375	-4.8992	-379.3563	-1852.7379	-4.8990	-0.0003	0.0004	-0.0002
2389200.5	-301.2941	-1764.1709	-5.2982	-301.2943	-1764.1705	-5.2985	-0.0001	-0.0004	0.0002
2389600.5	-252.4576	-1645.5282	-4.9561	-252.4570	-1645.5281	-4.9562	-0.0006	-0.0001	0.0001
239000.5	-246.5542	-1516.2720	-3.8280	-246.5541	-1516.2715	-3.8280	-0.0001	-0.0005	-0.0001
2390400.5	-283.1904	-1412.3702	-2.1639	-283.1902	-1402.3705	-2.1629	-0.0002	0.0004	-0.0001
2390800.5	-348.4372	-1327.6132	-0.4850	-348.4372	-1327.6127	-0.4845	0.0000	-0.0004	-0.0005
2391200.5	-414.0385	-1301.8133	0.6181	-414.0381	-1301.8138	0.6182	-0.0004	0.0006	-0.0001
2391600.5	-452.6357	-1315.9124	0.7805	-452.6359	-1315.9127	0.7801	0.0001	0.0003	0.0004
239200.5	-448.3734	-1348.2787	-0.0555	-448.3731	-1348.2789	-0.0556	-0.0003	0.0001	0.0001
2392400.5	-399.3374	-1374.8216	-1.4586	-399.3377	-1374.8214	-1.4587	-0.0004	-0.0002	0.0001
2392800.5	-314.3131	-1375.5577	-3.1550	-314.3131	-1375.5575	-3.1550	0.0001	-0.0002	0.0000
2393200.5	-208.9949	-1337.1812	-4.6442	-208.9938	-1337.1816	-4.6440	-0.0002	0.0004	-0.0001
2393600.5	-103.4344	-1254.2122	-5.5317	-103.4344	-1254.2126	-5.5319	-0.0000	0.0003	0.0002
239400.5	-19.7639	-1130.5551	-5.5626	-19.7638	-1130.5553	-5.5629	-0.0001	0.0002	0.0004
2394400.5	22.0263	-981.2997	-4.7157	22.0265	-981.3003	-4.7159	0.0002	0.0003	0.0002
2394800.5	11.5254	-831.9598	-3.2781	11.5254	-831.9598	-3.2779	-0.0000	-0.0003	-0.0003
2395200.5	-44.6816	-711.2109	-1.7931	-44.6818	-711.2104	-1.7927	0.0002	-0.0004	-0.0004
2395600.5	-123.5193	-638.3197	-0.8251	-123.5102	-638.3199	-0.8252	-0.0001	0.0001	0.0001
23960.5	-196.1179	-614.8328	-0.6848	-196.1181	-614.8324	-0.6846	0.0001	-0.0004	-0.0002
2396400.5	-246.3348	-627.3388	-1.3297	-240.0348	-627.0384	-1.3292	0.0000	-0.0003	-0.0005
2396800.5	-244.4201	-654.2190	-2.4573	-244.4203	-654.2181	-2.4573	0.0002	-0.0009	-0.0000
239700.5	-269.3405	-675.4089	-3.6504	-209.3403	-675.4091	-3.6504	-0.0002	0.0003	-0.0000
2397600.5	-143.4449	-672.9684	-4.4838	-143.4447	-672.9687	-4.4836	-0.0002	0.0003	-0.0002
239800.5	-62.2878	-634.6943	-4.6079	-62.2885	-634.6944	-4.6078	0.0007	0.0002	-0.0001
2398400.5	13.1161	-556.4196	-3.8325	13.1155	-556.4194	-3.8325	0.0005	-0.0001	-0.0000
2398800.5	59.6603	-445.4584	-2.2208	59.6602	-445.4585	-2.2209	0.0001	0.0002	0.0001
2399200.5	59.6591	-322.3513	-0.1491	59.6599	-322.3526	-0.1490	-0.0008	0.0013	-0.0001
2399600.5	10.5927	-216.0066	1.7808	10.5921	-216.0067	1.7811	0.0005	0.0001	-0.0003
240000.5	-71.3194	-151.4737	3.0068	-71.3186	-151.4731	3.0007	-0.0008	-0.0006	0.0002
2400400.5	-158.8668	-138.9942	3.2338	-158.8668	-138.9937	3.2336	0.0000	-0.0005	0.0002
2400800.5	-225.9783	-172.6107	2.5577	-225.9781	-172.6108	2.5574	-0.0002	0.0001	0.0003
2401200.5	-255.3546	-235.9092	1.2960	-255.0540	-235.9088	1.2955	-0.0006	-0.0004	0.0005
2401600.5	-238.5294	-308.2112	-0.1230	-238.5296	-308.2115	-0.1233	0.0002	0.0003	0.0003
2402000.5	-178.1166	-368.5564	-1.2695	-178.1167	-368.5563	-1.2698	0.0001	-0.0004	0.0003
2402400.5	-84.2058	-398.3669	-1.7943	-84.2063	-398.3667	-1.7947	0.0005	-0.0001	0.0004
2402800.5	24.3664	-384.7009	-1.5137	24.3868	-384.7013	-1.5138	-0.0003	0.0005	0.0002
2403200.5	122.3245	-325.0976	-0.5033	122.3243	-325.0976	-0.5035	0.0002	0.0000	0.0002
2403600.5	184.0592	-232.0792	0.8532	184.0595	-232.0789	0.8531	-0.0007	-0.0001	0.0001
240400.5	196.7275	-131.6202	1.9788	196.7270	-131.6205	1.9789	0.0005	0.0003	-0.0002
2404400.5	164.5332	-52.4626	2.3764	164.5331	-52.4628	2.3766	0.0001	0.0002	-0.0002
2404800.5	108.4849	-13.4220	1.8624	108.4846	-13.4222	1.8627	0.0003	0.0002	-0.0003
2405200.5	53.7162	-18.3785	0.5923	53.7162	-18.3794	0.5926	0.0000	0.0008	-0.0003
2405600.5	21.3686	-59.0703	-1.0641	21.3683	-59.0697	-1.0637	0.0003	-0.0006	-0.0004
240600.5	25.1371	-119.9281	-2.6598	25.1366	-119.9282	-2.6595	0.0005	0.0001	-0.0004
2406400.5	70.5326	-181.6650	-3.7689	70.5328	-181.6655	-3.7691	-0.0012	0.0005	0.0002
2406800.5	154.2266	-223.7945	-4.0661	154.2267	-223.7950	-4.0662	-0.0001	0.0005	0.0001
2407200.5	262.7918	-227.9141	-3.4115	262.7924	-227.9139	-3.4113	-0.0005	-0.0002	-0.0003
2407600.5	372.6550	-183.3845	-1.9454	372.6544	-183.3845	-1.9454	0.0006	0.0001	0.0000
240800.5	455.0426	-94.2906	-0.1180	455.0426	-94.2902	-0.1180	0.0000	-0.0004	-0.0000
2408400.5	487.5102	18.4531	1.4602	487.5103	18.4537	1.4604	-0.0003	-0.0006	-0.0001
2408800.5	464.9351	125.2589	2.3059	464.9351	125.2589	2.3058	-0.0001	-0.0000	0.0001
2409200.5	400.1339	200.9441	2.2658	400.1339	200.9445	2.2658	-0.0000	-0.0004	0.0000
2409600.5	315.3936	232.3786	1.5071	315.3933	232.3780	1.5071	0.0003	0.0006	0.0000
241000.5	233.8726	218.6054	0.3830	233.8727	218.6058	0.3833	-0.0001	-0.0004	-0.0003
2410400.5	175.0121	167.9597	-0.6933	175.0124	167.9592	-0.6928	-0.0002	0.0005	-0.0005
2410800.5	152.5088	95.5456	-1.3373	152.5089	95.5456	-1.3373	-0.0000	-0.0000	0.0000
2411200.5	172.4470	21.4669	-1.2711	172.4467	21.4673	-1.2714	0.0004	-0.0004	0.0003
2411600.5	230.4837	-31.8642	-0.4101	230.4839	-31.8639	-0.4105	-0.0003	-0.0002	0.0004
2412000.5	309.4799	-45.5572	1.0451	309.4794	-45.5573	1.0451	0.0004	0.0001	0.0000
2412400.5	381.9772	-13.1078	2.6051	381.9774	-13.1075	2.6059	-0.0001	-0.0002	-0.0008
2412800.5	420.7834	53.8238	3.6754	420.7832	53.8240	3.6758	0.0001	-0.0002	-0.0004
2413200.5	411.8583	130.1376	3.8436	411.8583	130.1370	3.8432	-0.0000	0.0006	0.0004
2413600.5	358.8369	189.4452	3.0478	358.8368	189.4460	3.0476	0.0001	-0.0008	0.0002
241400.5	277.8158	213.3360	1.5320	277.8163	213.3356	1.5322	-0.0005	0.0005	-0

Table C-4 (Continued)

Julian Day	$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\gamma \cdot 10^6$	$\alpha_r \cdot 10^6$	$\beta_r \cdot 10^6$	$\gamma_r \cdot 10^6$	$(\alpha - \alpha_r) \cdot 10^6$	$(\beta - \beta_r) \cdot 10^6$	$(\gamma - \gamma_r) \cdot 10^6$
2415200.5	73.3402	40.6280	-3.1211	73.3405	40.8280	-3.1212	-0.0004	-0.0000	0.0001
2415600.5	75.6669	-65.0369	-3.4271	75.6672	-65.0372	-3.4273	-0.0002	0.0003	0.0003
2416000.5	125.9840	-160.6429	-2.8330	125.9843	-160.6430	-2.8334	-0.0003	0.0002	0.0004
2416400.5	213.8981	-221.0459	-1.5748	213.8974	-221.0458	-1.5748	0.0007	-0.0001	0.0001
2416800.5	314.7803	-230.1430	-0.1667	314.7800	-230.1432	-0.1662	0.0003	0.0002	-0.0005
2417200.5	398.2814	-189.4528	0.8126	398.2801	-189.4527	0.8130	0.0013	-0.0000	-0.0004
2417600.5	441.8810	-117.4095	1.033	441.8811	-117.4095	1.0336	-0.0002	-0.0000	-0.0002
2418000.5	438.7030	-39.5103	0.3956	438.7030	-39.5103	0.3959	0.0000	-0.0003	-0.0003
2418400.5	395.9501	21.7894	-0.7431	395.8998	21.7892	-0.7434	0.0002	0.0002	0.0003
2418800.5	328.9705	51.5795	-2.1116	328.9702	51.5799	-2.0119	0.0002	-0.0003	0.0003
2419200.5	257.1180	43.3838	-2.9864	257.0181	43.3833	-2.9861	-0.0001	0.0005	-0.0003
2419600.5	159.6130	-0.7187	-3.3041	159.8139	-0.7189	-3.3036	-0.0009	0.0002	-0.0005
2420000.5	175.0263	-69.4976	-2.7447	175.0267	-69.4974	-2.7444	-0.0004	-0.0002	-0.0003
2420400.5	193.7311	-142.4703	-1.3256	193.7319	-142.4700	-1.3260	-0.0008	-0.0003	0.0003
2420800.5	254.1724	-193.3288	0.6261	254.1719	-193.3291	0.6257	0.0005	0.0002	0.0005
2421200.5	338.1597	-199.4227	2.5316	338.1595	-199.4227	2.5314	0.0002	-0.0001	0.0002
2421600.5	416.6658	-153.1748	3.8005	416.5650	-153.1748	3.8003	0.0008	-0.0000	0.0002
2422000.5	462.3811	-65.5460	4.1059	462.3811	-65.5460	4.1060	0.0003	-0.0001	-0.0001
2422400.5	460.9322	41.4673	3.4798	460.9021	41.0478	3.4802	0.0001	-0.0004	-0.0003
2422800.5	411.4128	142.4118	2.2188	411.4129	142.4113	2.2188	-0.0001	0.0005	-0.0000
2423200.5	323.8121	218.4426	0.7404	323.8125	218.4425	0.7402	-0.0004	0.0002	0.0002
2423600.5	214.7724	255.4743	-0.5225	214.7724	255.4738	-0.5229	-0.0001	0.0004	0.0003
2424000.5	105.1272	247.6348	-1.2041	105.0274	247.6351	-1.2042	-0.0003	-0.0003	0.0001
2424400.5	16.8418	196.7371	-1.1873	16.8415	198.7370	-1.0869	0.0002	0.0001	-0.0005
2424800.5	-30.4666	124.2294	-0.1999	-30.4663	124.2295	-0.1997	-0.0002	-0.0001	-0.0002
2425200.5	-27.9742	50.1097	1.1214	-27.9747	50.1093	1.1211	0.0005	0.0004	0.0003
2425600.5	16.0425	4.8087	2.3133	16.1431	4.8085	2.3132	-0.0007	0.0002	0.0001
2426000.5	77.6043	6.2465	2.8463	77.6045	6.2468	2.8467	-0.0002	-0.0003	-0.0004
2426400.5	128.7339	54.2983	2.4819	128.7038	54.2981	2.4824	0.0000	0.0002	-0.0005
2426800.5	148.1126	134.1682	1.3306	148.1127	134.1694	1.3305	-0.0001	-0.0012	0.0000
2427200.5	126.1274	224.7829	-0.2630	126.1275	224.7834	-0.2633	-0.0001	-0.0005	0.0003
2427600.5	66.2573	305.1752	-1.8605	66.2570	305.1760	-1.8602	0.0002	-0.0008	-0.0003
2428000.5	-22.9594	357.7161	-3.6301	-22.9993	357.7159	-3.0296	-0.0001	0.0002	-0.0005
2428400.5	-123.9427	370.2555	-3.4268	-123.9422	370.2559	-3.4266	-0.0005	-0.0003	-0.0002
2428800.5	-214.3695	339.1997	-2.8764	-214.3697	339.0998	-2.8765	0.0002	-0.0001	0.0000
2429200.5	-270.1638	272.8503	-1.4710	-270.7640	272.8502	-1.4712	0.0002	0.0001	0.0002
2429600.5	-276.1934	194.0573	0.3838	-276.1939	194.0575	0.3841	0.0005	-0.0002	-0.0003
2430000.5	-229.7214	133.3899	2.0840	-229.7215	133.3895	2.0844	-0.0003	0.0003	-0.0004
2430400.5	-148.7970	116.4123	3.1030	-148.7975	116.4123	3.1029	0.0005	0.0000	0.0001
2430800.5	-60.6712	152.9427	3.2271	-60.6706	152.9426	3.2272	-0.0005	-0.0002	-0.0001
2431200.5	9.3151	236.6942	2.5803	9.3153	236.6948	2.5803	-0.0001	-0.0006	0.0000
2431600.5	44.1935	351.4346	1.4949	44.1934	351.4347	1.4948	0.0001	-0.0001	0.0001
2432000.5	36.6222	476.7183	0.3784	36.6223	476.9185	0.3786	-0.0001	-0.0002	-0.0002
2432400.5	-11.6286	592.4182	-0.3772	-11.6284	592.4378	-0.3772	-0.0002	0.0004	0.0000
2432800.5	-90.0311	679.3567	-0.4710	-90.0295	679.3569	-0.4710	-0.0006	-0.0002	-0.0001
2433200.5	-179.6246	724.7852	0.2226	-179.6246	724.7849	0.2233	0.0000	0.0003	-0.0007
2433600.5	-255.4665	727.1392	1.5598	-255.4660	727.1392	1.5602	-0.0005	-0.0000	0.0004
2434000.5	-294.1673	700.7613	3.0948	-294.1078	700.7609	3.0946	0.0005	0.0004	0.0002
2434400.5	-244.7867	673.1618	4.2314	-244.7871	673.1616	4.2312	0.0004	0.0002	0.0002
2434800.5	-235.4932	672.8723	4.5126	-235.4933	672.8720	4.5124	0.0001	0.0002	0.0003
2435200.5	-167.7769	716.9403	3.8204	-167.7777	716.9399	3.8202	-0.0003	0.0004	0.0002
2435600.5	-106.1588	807.2151	2.3601	-106.1591	807.2154	2.3602	0.0003	-0.0003	-0.0001
2436000.5	-70.7730	933.8472	0.5243	-70.7728	933.8464	0.5243	-0.0002	0.0008	0.0000
2436400.5	-74.4902	1080.0661	-1.2388	-74.4904	1080.0660	-1.2391	0.0002	0.0000	0.0003
2436800.5	-122.6678	1225.5127	-2.5143	-122.0675	1225.5127	-2.5143	-0.0003	-0.0004	-0.0001
2437200.5	-209.1143	1348.6505	-2.9939	-209.1145	1348.6508	-2.9938	0.0002	-0.0003	-0.0001
2437600.5	-320.8454	1430.4105	-2.5685	-320.8465	1430.4104	-2.5689	0.0012	0.0003	0.0004
2438000.5	-432.6891	1460.6638	-1.4219	-432.6888	1460.6635	-1.4219	-0.0002	0.0002	-0.0000
2438400.5	-516.4422	1445.3929	-0.0303	-516.4424	1445.3924	-0.0302	0.0003	0.0004	-0.0001
2438800.5	-552.1648	1474.4749	1.1111	-552.1646	1474.4747	1.0169	-0.0001	0.0001	0.0002
2439200.5	-537.4367	1376.6871	1.3089	-537.4366	1376.6869	1.3090	-0.0002	0.0002	-0.0001
2439600.5	-486.3123	1376.8224	0.7806	-486.3125	1376.8223	0.7801	0.0002	0.0000	0.0004
2440000.5	-419.3146	1418.8868	-0.3369	-419.3146	1418.8873	-0.3372	0.0000	-0.0005	0.0003
2440400.5	-358.8556	1501.9192	-1.6534	-358.8459	1501.9191	-1.6532	0.0003	0.0001	-0.0002
2440800.5	-322.2665	1616.0722	-2.7434	-322.2664	1616.0717	-2.7440	-0.0001	0.0003	0.0006
2441200.5	-321.4146	1745.1485	-3.2378	-321.4149	1745.1483	-3.2378	0.0003	0.0002	-0.0000
2441600.5	-360.1434	1868.4761	-2.8910	-360.1432	1868.4762	-2.8906	-0.0003	0.0001	-0.0004
2442000.5	-431.4348	1963.9368	-1.6766	-431.4353	1963.9365	-1.6768	0.0005	0.0003	0.0002
2442400.5	-515.7699	2014.4709	0.1290	-515.7700	2014.4710	0.1293	0.0000	-0.0001	-0.0003
2442800.5	-565.4537	2017.1509	1.9792	-565.4541	2017.1509	1.9792	0.0005	0.0001	-0.0000
2443200.5	-613.9684	1987.6333	3.2641	-613.9679	1987.6331	3.2641	-0.0005	-0.0001	-0.0001
2443600.5	-591.7499	1953.5376	3.6027	-591.7505	1953.5375	3.6037	0.0006	0.0001	-0.0003
2444000.5	-524.5713	1941.7739	2.9739	-524.5709	1941.7740	2.9738	-0.0004	-0.0001	0.0001
2444400.5	-429.712	1969.9774	1.6431	-429.7016	1969.9778	1.6430	0.0005	-0.0003	0.0001
2444800.5	-328.5166	2044.6344	0.0220	-328.5168	2044.6344	0.0222	0.0002	-0.0001	-0.0001
2445200.5	-242.0383	2162.3654	-1.4502	-242.0382	2162.3657	-1.4503	-0.0001	-0.0003	0.0001
2445600.5	-188.2764	2311.3563	-2.3934	-188.2760	2311.3566	-2.3936	-0.0004	-0.0003	0.0001
2446000.5	-179.3586	2472.3518	-2.5638	-179.3584	2472.3514	-2.5636	-0.0001	0.0004	-0.0002
2446400.5	-217.2992	2620.6710	-1.9449	-217.2987	2620.6714	-1.9449	-0.0005	-0.0004	-0.0001
2446800.5	-289.8813	2732.0125	-0.8238	-289.8811	2732.0125	-0.8235	-0.0002	0.0001	-0.0003
2447200.5	-371.5489	2792.6414	0.2623	-371.5487	2792.6413	0.2622	-0.0002	0.0001	0.0001
2447600.5	-433.1363	2807.5007	0.7555	-433.1362	2807.5003	0.7559	-0.0001	0.0004	-0.0004
2448000.5	-455.0590	2797.6909	0.3592	-455.0589	2797.6912	0.3588	-0.0001	-0.0003	0.0004
2448400.5	-433.4572	2789.4727	-0.8736	-433.4577	2789.4728	-0.8736	0.0005	-0.0001	0.0000
2448800.5	-377.4559	2804.5822	-2.6248	-377.4556	2804.5820	-2.6249	-0.0003	0.0002	0.0001
2449200.5	-303.5271	2856.2364	-4.4615	-303.5272	2856.2362	-4.4618	0.0000	0.0002	0.0004
2449600.5	-231.2527	2948.6402	-5.9457	-231.2529	2948.6403	-5.9458	0.0002	-0.0001	0.0001
2450000.5	-180.5978	3077.0687	-6.7144	-180.5983	3077.0691	-6.7139	0.0004	-0.0004	-0.0004
2450400.5	-168.9176	3227.5182	-6.5603	-168.9178	3227.5181	-6.5602	0.0002	0.0001	-0.0000
2450800.5	-205.9215	3377.1343	-5.5286	-205.9216	3377.1343	-5.5283	0.0001	0.0000	-0.0003
2									



Appendix D

Perturbations of the Five Outer Planets by the Four Inner Ones

These perturbations have been given by Clemence (Reference D-1). They are computed here in the components α, β, γ for convenience. The elliptic reference elements are slightly different, but this is of no consequence owing to the small magnitudes of the coefficients. The comparisons confirm the present computations.

The reference elements used for the outer planets are those given on page 17. For the inner planets, Newcomb's mean elements for 1900 referred to the ecliptic and mean equinox of 1950 are used. The values are given in Table D-1. The epoch is JD 2415200.5.

Table D-1

Reference Elements of the Inner Planets.

Planet	Semi-Major Axis, a , in a.u.	Eccentricity, e	Mean Motion in Degrees per Day, n	Longitude of Ascending Node, Ω	Argument of Perihelion, ω	Inclination, i	Mean Anomaly at Epoch, g_0
Mercury	0.3870 9860	0.2056 1421	4.0923 3882	47°801 383	28°796 508	7°006 781	120°946 538
Venus	0.7233 3162	0.0068 2069	1.6021 30491	76.202 356	54.659 325	3.394 547	141.787 773
Earth	1.0000 0022	0.0167 5104	0.9856 09134	353.951 667	107.967 350	0.006 542	176.378 294
Mars	1.5236 8838	0.0933 1290	0.5240 32950	49.318 297	285.598 386	1.854 119	54.117 372

The angular elements referred to the ecliptic and mean equinox of 1950 are taken from Reference D-2 with a 180 degree correction to the longitude of the ascending node of the earth. The values of the masses are taken from Reference D-3.

The equation of motion of the j^{th} planet may be written

$$\frac{d^2 \vec{r}_j}{dt^2} = -\mu_j^2 \frac{\vec{r}_j}{r_j^3} + \mu_j^2 \sum_{\substack{i=1 \\ i \neq j}}^9 \frac{m_i}{M+m_j} \left[\frac{\vec{\rho}_{ij}}{\rho_{ij}^3} - \frac{\vec{r}_i}{r_i^3} \right]$$

where

\vec{r}_j = the position vector of the j^{th} planet with respect to the sun

$\vec{\rho}_{ij} = \vec{r}_i - \vec{r}_j$

M = the mass of the Sun

m_j = the mass of the j^{th} planet

$\mu_j'^2 = k^2 (M + m_j)$ where k is the Gaussian constant

t = the time.

Putting

$$M' = M + m_1 + m_2 + m_3 + m_4$$

$$\mu_j'^2 = k^2 (M' + m_j)$$

and noticing that

$$\mu_j'^2 \frac{m_i}{M' + m_j} = \mu_j^2 \frac{m_i}{M + m_j},$$

the equation of motion can be written

$$\frac{d^2 \vec{r}_j}{dt^2} = -\mu_j'^2 \frac{\vec{r}_j}{r_j^3} + \mu_j^2 \sum_{i=1}^4 \frac{m_i}{M + m_j} \left[\frac{\vec{\rho}_{ij}}{r_{ij}^3} + \frac{\vec{r}_j}{r_j^3} - \frac{\vec{r}_i}{r_i^3} \right] + \mu_j'^2 \sum_{\substack{i=5 \\ i \neq j}}^9 \frac{m_i}{M' + m_j} \left[\frac{\vec{\rho}_{ij}}{r_{ij}^3} - \frac{\vec{r}_i}{r_i^3} \right].$$

The second term on the right hand side was omitted in the solution in Chebyshev series and is to be taken into account here. Due to the small values of m_i for $i \leq 4$, it is not necessary to consider terms of second or higher orders. The perturbations are computed using the usual first-order method with the modified expression for the disturbing force. All terms as large as $0.5 \cdot 10^{-10}$ radians are given. The time is expressed in centuries of 36525 Julian days.

The coefficients are in units of the sixth decimal of a radian. They are to be multiplied by $T^n \cos(g + jg')$ where T is the time in centuries, and g and g' are the mean anomalies of the disturbed and disturbing planets respectively.

Table D-2
Perturbations of Jupiter Due to Mercury.

i	j	n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
			cos	sin	cos	sin	cos	sin
0	0	1	-0.0000	0.	0.	0.	0.0003	0.
1	0	1	-0.0006	-0.0016	-0.0032	0.0013	-0.0040	-0.0014
2	0	1	-0.0000	-0.0000	-0.0000	0.0000	-0.0001	-0.0000
0	0	0	0.0003	0.	0.	0.	-0.0003	0.
1	0	0	-0.0009	-0.0017	-0.0000	0.0000	-0.0000	0.0000
2	0	0	-0.0001	-0.0001	-0.0001	0.0000	-0.0000	-0.0000
-1	-1	0	0.0001	-0.0001	0.0001	0.0001	-0.0000	-0.0000
0	-1	0	-0.0000	0.0000	-0.0008	0.0004	0.0008	-0.0010
1	-1	0	0.0056	0.0107	0.0107	-0.0056	0.0000	0.0000
2	-1	0	0.0003	0.0005	0.0003	-0.0001	0.0000	0.0000
0	-2	0	-0.0000	0.0000	-0.0001	0.0000	0.0001	-0.0001
1	-2	0	0.0006	0.0011	0.0011	-0.0006	-0.0000	-0.0000
2	-2	0	0.0000	0.0001	0.0000	-0.0000	-0.0000	0.0000
1	-3	0	0.0001	0.0002	0.0002	-0.0001	-0.0000	-0.0000

Table D-3
Perturbations of Jupiter Due to Venus.

i	j	n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
			cos	sin	cos	sin	cos	sin
0	0	1	-0.0001	0.	0.	0.	0.0037	0.
1	0	1	-0.0018	-0.0961	-0.1923	0.0035	-0.0508	-0.0580
2	0	1	-0.0000	-0.0023	-0.0023	0.0000	-0.0012	-0.0014
3	0	1	-0.0000	-0.0001	-0.0001	0.0000	-0.0000	-0.0001
0	0	0	0.0122	0.	0.	0.	-0.0002	0.
1	0	0	0.0026	-0.0016	-0.0000	0.0000	-0.0000	0.0000
2	0	0	0.0002	-0.0001	-0.0000	-0.0001	0.0001	-0.0001
-1	-1	0	0.0002	-0.0000	0.0000	0.0002	-0.0000	0.0000
0	-1	0	-0.0000	0.0000	-0.0221	-0.0112	0.0125	-0.0050
1	-1	0	-0.1539	0.3039	0.3039	0.1539	-0.0000	0.0000
2	-1	0	-0.0075	0.0147	0.0074	0.0037	0.0000	0.0000
3	-1	0	-0.0004	0.0008	0.0003	0.0001	0.0000	0.0000
0	-2	0	-0.0000	-0.0000	-0.0001	-0.0000	0.0000	-0.0000
1	-2	0	-0.0005	0.0010	0.0010	0.0005	0.0000	-0.0000
2	-2	0	-0.0001	-0.0000	0.0001	-0.0000	0.0000	-0.0000

Table D-4
Perturbations of Jupiter Due to Earth.

i	j	n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
			cos	sin	cos	sin	cos	sin
0	0	1	-0.0005	0.	0.	0.	-0.0005	0.
1	0	1	-0.0192	-0.2325	-0.4650	0.0383	0.0075	0.1093
2	0	1	-0.0005	-0.0056	-0.0056	0.0005	0.0002	0.0027
3	0	1	-0.0000	-0.0002	-0.0001	0.0000	0.0000	0.0001
0	0	0	0.0294	0.	0.	0.	0.0002	0.
1	0	0	0.0041	-0.0079	0.0000	0.0000	0.0000	-0.0000
2	0	0	0.0003	-0.0006	-0.0002	-0.0003	-0.0001	0.0000
-1	-1	0	-0.0000	-0.0003	0.0003	0.0000	0.0000	-0.0000
0	-1	0	-0.0000	0.0000	-0.0425	0.0016	-0.0004	0.0133
1	-1	0	0.0214	0.5832	0.5828	-0.0214	-0.0000	-0.0000
2	-1	0	0.0012	0.0283	0.0141	-0.0004	-0.0000	-0.0000
3	-1	0	0.0001	0.0015	0.0005	0.0000	-0.0000	-0.0000
4	-1	0	0.0000	0.0001	0.0000	0.0000	-0.0000	-0.0000
0	-2	0	-0.0000	0.0000	-0.0004	0.0000	-0.0000	0.0001
1	-2	0	0.0002	0.0049	0.0049	-0.0002	0.0000	-0.0000
2	-2	0	-0.0006	0.0003	0.0001	-0.0004	0.0000	-0.0000
3	-2	0	-0.0001	0.0000	-0.0000	-0.0001	0.0000	-0.0000
1	-3	0	0.0000	0.0001	0.0001	-0.0000	0.0000	0.0000
3	-3	0	-0.0000	-0.0001	0.0000	-0.0000	0.0000	0.0000

Table D-5
Perturbations of Jupiter Due to Mars.

i	j	n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
			cos	sin	cos	sin	cos	sin
0	0	1	0.0007	0.	0.	0.	0.0024	0.
1	0	1	0.0279	-0.0296	-0.0592	-0.0558	-0.0325	0.0047
2	0	1	0.0007	-0.0007	-0.0007	-0.0007	-0.0008	0.0001
0	0	0	0.0078	0.	0.	0.	0.0002	0.
1	0	0	-0.0049	0.0050	0.0000	0.0000	0.0000	0.0000
2	0	0	-0.0003	0.0002	0.0002	0.0001	0.0000	-0.0000
-1	-1	0	0.0001	0.0001	-0.0001	0.0001	0.0000	-0.0000
0	-1	0	-0.0001	-0.0000	0.0043	0.0053	-0.0012	-0.0020
1	-1	0	0.0735	-0.0597	-0.0592	-0.0729	-0.0000	0.0000
2	-1	0	0.0034	-0.0016	-0.0027	-0.0020	-0.0000	-0.0000
3	-1	0	0.0000	0.0003	-0.0004	-0.0003	-0.0000	-0.0000
4	-1	0	-0.0001	0.0001	-0.0001	-0.0001	0.0000	-0.0000
5	-1	0	-0.0001	0.0001	-0.0001	-0.0001	0.0000	-0.0000
6	-1	0	0.0000	-0.0000	0.0001	0.0001	-0.0000	0.0000
0	-2	0	-0.0000	-0.0000	0.0002	0.0002	-0.0001	-0.0001
1	-2	0	0.0034	-0.0028	-0.0028	-0.0034	0.0000	0.0000
2	-2	0	0.0003	-0.0008	0.0004	0.0000	0.0000	0.0000
3	-2	0	0.0001	-0.0001	0.0000	0.0001	-0.0000	0.0000
1	-3	0	0.0002	-0.0002	-0.0002	-0.0002	0.0000	0.0000
3	-3	0	-0.0000	-0.0001	0.0001	-0.0000	0.0000	0.0000

Table D-6
Perturbations of Saturn Due to Mercury.

i	j	n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
			cos	sin	cos	sin	cos	sin
1	0	1	-0.0001	-0.0002	-0.0004	0.0002	-0.0002	0.0005
0	0	0	0.0001	0.	0.	0.	-0.0002	0.
1	0	0	-0.0010	0.0003	0.0000	0.0000	0.0000	0.0000
2	0	0	-0.0001	0.0000	0.0000	0.0000	-0.0000	-0.0000
0	-1	0	0.0000	-0.0000	0.0001	0.0005	0.0006	-0.0005
1	-1	0	0.0064	-0.0018	-0.0018	-0.0064	0.0000	-0.0000
2	-1	0	0.0003	-0.0001	-0.0000	-0.0002	-0.0000	0.0000
0	-2	0	0.0000	-0.0000	0.0000	0.0001	0.0001	-0.0000
1	-2	0	0.0006	-0.0002	-0.0002	-0.0006	-0.0000	0.0000
1	-3	0	0.0001	-0.0000	-0.0000	-0.0001	-0.0000	0.0000

Table D-7
Perturbations of Saturn Due to Venus.

i	j	n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
			cos	sin	cos	sin	cos	sin
0	0	1	-0.0000	0.	0.	0.	0.0003	0.
1	0	1	-0.0002	-0.0127	-0.0254	0.0004	-0.0038	0.0073
2	0	1	-0.0000	-0.0003	-0.0003	0.0000	-0.0001	0.0002
0	0	0	0.0036	0.	0.	0.	-0.0000	0.
1	0	0	-0.0002	-0.0006	-0.0000	0.0000	0.0000	0.0000
2	0	0	-0.0000	-0.0001	-0.0000	-0.0000	-0.0000	-0.0000
-1	-1	0	-0.0001	-0.0000	0.0000	-0.0001	-0.0000	-0.0000
0	-1	0	0.0000	0.0000	-0.0094	0.0118	0.0066	0.0013
1	-1	0	0.1454	0.1158	0.1158	-0.1454	0.0000	0.0000
2	-1	0	0.0079	0.0063	0.0031	-0.0039	-0.0000	-0.0000
3	-1	0	0.0005	0.0004	0.0001	-0.0002	-0.0000	-0.0000
1	-2	0	0.0005	0.0004	0.0004	-0.0005	-0.0000	-0.0000

Table D-8

Perturbations of Saturn Due to Earth.

i j n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
	cos	sin	cos	sin	cos	sin
0 0 1	-0.0000	0.	0.	0.	-0.0018	0.
1 0 1	-0.0004	-0.0286	-0.0573	0.0008	0.0222	0.0086
2 0 1	-0.0000	-0.0008	-0.0008	0.0000	0.0006	0.0002
0 0 0	0.0086	0.	0.	0.	-0.0000	0.
1 0 0	-0.0026	-0.0007	0.0000	0.0000	0.0000	-0.0000
2 0 0	-0.0002	-0.0000	-0.0000	0.0000	-0.0000	0.0001
-1 -1 0	0.0000	-0.0001	0.0001	0.0000	0.0000	-0.0000
0 -1 0	-0.0000	0.0000	-0.0043	0.0255	0.0028	0.0135
1 -1 0	0.3137	0.0528	0.0528	-0.3137	0.0000	0.0000
2 -1 0	0.0170	0.0029	0.0014	-0.0085	-0.0000	0.0000
3 -1 0	0.0010	0.0002	0.0001	-0.0003	-0.0000	0.0000
4 -1 0	0.0001	0.0000	0.0000	-0.0000	-0.0000	0.0000
0 -2 0	0.0000	0.0000	-0.0000	0.0002	0.0000	0.0001
1 -2 0	0.0026	0.0004	0.0004	-0.0026	-0.0000	-0.0000
2 -2 0	0.0002	0.0000	0.0000	-0.0001	-0.0000	-0.0000

Table D-9

Perturbations of Saturn Due to Mars.

i j n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
	cos	sin	cos	sin	cos	sin
0 0 1	0.0001	0.	0.	0.	-0.0002	0.
1 0 1	0.0023	-0.0089	-0.0177	-0.0046	0.0023	0.0052
2 0 1	0.0001	-0.0002	-0.0002	-0.0001	0.0001	0.0001
0 0 0	0.0021	0.	0.	0.	0.0000	0.
1 0 0	0.0021	0.0034	0.0000	-0.0000	0.0000	-0.0000
2 0 0	0.0001	0.0003	0.0001	-0.0001	-0.0000	0.0000
-1 -1 0	-0.0000	0.0001	-0.0001	-0.0000	0.0000	0.0000
0 -1 0	-0.0000	-0.0000	0.0037	-0.0019	-0.0001	-0.0021
1 -1 0	-0.0237	-0.0456	-0.0456	0.0237	-0.0000	0.0000
2 -1 0	-0.0013	-0.0025	-0.0012	0.0007	0.0000	0.0000
3 -1 0	-0.0001	-0.0002	-0.0000	0.0000	0.0000	0.0000
0 -2 0	-0.0000	0.0000	0.0002	-0.0001	-0.0000	-0.0001
1 -2 0	-0.0011	-0.0021	-0.0021	0.0011	0.0000	-0.0000
2 -2 0	-0.0001	-0.0001	-0.0001	0.0000	0.0000	-0.0000
1 -3 0	-0.0001	-0.0001	-0.0001	0.0001	0.0000	-0.0000

Table D-10
Perturbations of Uranus Due to Mercury.

i	j	n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
			cos	sin	cos	sin	cos	sin
1	0	1	-0.0000	0.0001	0.0002	0.0001	0.0000	0.0000
0	0	0	0.0000	0.	0.	0.	-0.0001	0.
1	0	0	0.0000	0.0005	0.0000	-0.0000	0.0000	-0.0000
0	-1	0	-0.0000	-0.0000	0.0002	-0.0000	0.0002	-0.0003
1	-1	0	-0.0002	-0.0033	-0.0033	0.0002	-0.0000	-0.0000
2	-1	0	-0.0000	-0.0002	-0.0001	0.0000	0.0000	-0.0000
1	-2	0	-0.0000	-0.0003	-0.0003	0.0000	0.0000	0.0000
1	-3	0	-0.0000	-0.0001	-0.0001	0.0000	-0.0000	-0.0000

Table D-11
Perturbations of Uranus Due to Venus.

i	j	n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
			cos	sin	cos	sin	cos	sin
1	0	1	-0.0006	-0.0009	-0.0018	0.0012	0.0000	0.0009
0	0	0	0.0009	0.	0.	0.	-0.0000	0.
1	0	0	-0.0002	0.0002	0.0000	0.0000	0.0000	-0.0000
-1	-1	0	-0.0001	0.0000	-0.0000	-0.0001	0.0000	0.0000
0	-1	0	0.0000	-0.0000	0.0042	0.0051	0.0034	-0.0025
1	-1	0	0.0716	-0.0584	-0.0584	-0.0716	0.0000	-0.0000
2	-1	0	0.0034	-0.0028	-0.0014	-0.0017	0.0000	-0.0000
3	-1	0	0.0002	-0.0001	-0.0000	-0.0001	-0.0000	0.0000
1	-2	0	0.0002	-0.0002	-0.0002	-0.0002	-0.0000	0.0000

Table D-12

Perturbations of Uranus Due to Earth.

i j n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
	cos	sin	cos	sin	cos	sin
1 0 1	-0.0002	-0.0019	-0.0039	0.0004	-0.0001	-0.0006
0 0 0	0.0021	0.	0.	0.	0.0000	0.
1 0 0	-0.0005	0.0018	0.0000	0.0000	-0.0000	0.0000
2 0 0	-0.0000	0.0001	0.0000	-0.0000	0.0000	-0.0000
-1 -1 0	-0.0000	0.0001	-0.0001	-0.0000	-0.0000	-0.0000
0 -1 0	0.0000	-0.0000	0.0104	0.0042	-0.0010	0.0019
1 -1 0	0.0589	-0.1468	-0.1468	-0.0589	0.0000	0.0000
2 -1 0	0.0028	-0.0069	-0.0035	-0.0014	0.0000	-0.0000
3 -1 0	0.0001	-0.0004	-0.0001	-0.0000	0.0000	-0.0000
0 -2 0	0.0000	-0.0000	0.0001	0.0000	-0.0000	0.0000
1 -2 0	0.0005	-0.0012	-0.0012	-0.0005	-0.0000	-0.0000
2 -2 0	0.0000	-0.0001	-0.0000	-0.0000	-0.0000	-0.0000

Table D-13

Perturbations of Uranus Due to Mars.

i j n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
	cos	sin	cos	sin	cos	sin
1 0 1	-0.0000	-0.0007	-0.0013	0.0000	0.0002	0.0002
0 0 0	0.0005	0.	0.	0.	0.0001	0.
1 0 0	0.0018	-0.0005	-0.0000	-0.0000	0.0000	-0.0000
2 0 0	0.0001	-0.0000	-0.0000	-0.0000	-0.0000	0.0000
0 -1 0	-0.0000	0.0000	-0.0005	-0.0018	-0.0005	-0.0003
1 -1 0	-0.0247	0.0067	0.0067	0.0247	0.0000	0.0000
2 -1 0	-0.0012	0.0003	0.0002	0.0006	0.0000	-0.0000
3 -1 0	-0.0001	0.0000	0.0000	0.0000	0.0000	-0.0000
0 -2 0	-0.0000	0.0000	-0.0030	-0.0001	-0.0000	-0.0000
1 -2 0	-0.0011	0.0003	0.0003	0.0011	-0.0000	-0.0000
2 -2 0	-0.0001	0.0000	0.0000	0.0000	-0.0000	-0.0000
1 -3 0	-0.0001	0.0000	0.0000	0.0001	-0.0000	-0.0000

Table D-14
Perturbations of Neptune Due to Mercury.

i j n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
	cos	sin	cos	sin	cos	sin
1 0 1	-0.0001	-0.0001	-0.0002	0.0003	-0.0000	0.0000
0 0 0	0.0000	0.	0.	0.	-0.0001	0.
1 0 0	-0.0003	-0.0002	-0.0000	0.0000	0.0000	0.0000
0 -1 0	0.0000	0.0000	-0.0000	0.0000	0.0002	-0.0002
1 -1 0	0.0018	0.0011	0.0011	-0.0018	0.0000	0.0000
1 -2 0	0.0002	0.0001	0.0001	-0.0002	0.0000	-0.0000

Table D-15
Perturbations of Neptune Due to Venus.

i j n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
	cos	sin	cos	sin	cos	sin
1 0 1	0.0004	-0.0007	-0.0014	-0.0008	-0.0002	0.0000
0 0 0	0.0004	0.	0.	0.	-0.0000	0.
1 0 0	0.0001	-0.0002	-0.0000	0.0000	0.0000	0.0000
0 -1 0	0.0000	0.0000	-0.0008	0.0001	0.0029	-0.0002
1 -1 0	0.0055	0.0587	0.0587	-0.0055	0.0000	-0.0000
2 -1 0	0.0000	0.0005	0.0003	-0.0000	0.0000	-0.0000
1 -2 0	0.0000	0.0002	0.0002	-0.0000	-0.0000	-0.0000

Table D-16
Perturbations of Neptune Due to Earth.

i j n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
	cos	sin	cos	sin	cos	sin
1 0 1	-0.0000	-0.0005	-0.0010	0.0001	0.0000	0.0003
0 0 0	0.0008	0.	0.	0.	-0.0000	0.
1 0 0	-0.0006	-0.0011	-0.0000	0.0000	0.0000	-0.0000
0 -1 0	0.0000	0.0000	-0.0011	0.0008	0.0015	0.0027
1 -1 0	0.0570	0.0833	0.0833	-0.0570	0.0000	0.0000
2 -1 0	0.0005	0.0007	0.0004	-0.0003	0.0000	0.0000
1 -2 0	0.0005	0.0007	0.0007	-0.0005	0.0000	-0.0000

Table D-17
Perturbations of Neptune Due to Mars.

i j n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
	cos	sin	cos	sin	cos	sin
1 0 1	-0.0000	-0.0000	-0.0000	0.0000	-0.0001	0.0001
0 0 0	0.0002	0.	0.	0.	0.0000	0.
1 0 0	-0.0004	0.0011	0.0000	0.0000	0.0000	0.0000
0 -1 0	0.0000	-0.0000	0.0002	0.0001	-0.0003	-0.0006
1 -1 0	0.0052	-0.0154	-0.0154	-0.0052	-0.0000	0.0000
2 -1 0	0.0000	-0.0001	-0.0001	-0.0000	-0.0000	0.0000
1 -2 0	0.0002	-0.0007	-0.0007	-0.0002	0.0000	-0.0000
1 -3 0	0.0000	-0.0001	-0.0001	-0.0000	0.0000	-0.0000

Table D-18
Perturbations of Pluto Due to Mercury.

i j n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
	cos	sin	cos	sin	cos	sin
1 0 1	0.0001	0.0002	0.0004	-0.0003	0.0000	0.0000
0 0 0	-0.0000	0.	0.	0.	-0.0001	0.
1 0 0	0.0001	0.0002	-0.0000	-0.0000	0.0000	-0.0000
2 0 0	0.0001	0.0001	0.0000	-0.0000	0.0000	0.0000
0 -1 0	-0.0000	-0.0000	0.0003	-0.0005	0.0003	0.0002
1 -1 0	-0.0013	-0.0009	-0.0009	0.0013	-0.0000	-0.0000
2 -1 0	-0.0003	-0.0002	-0.0001	0.0002	0.0000	0.0000
3 -1 0	-0.0001	-0.0001	-0.0000	0.0000	0.0000	0.0000
0 -2 0	0.0000	-0.0000	0.0000	-0.0001	0.0000	0.0000
1 -2 0	-0.0001	-0.0001	-0.0001	0.0001	0.0000	-0.0000

Table D-19
Perturbations of Pluto Due to Venus.

i j n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
	cos	sin	cos	sin	cos	sin
0 0 1	-0.0001	0.	0.	0.	0.0000	0.
1 0 1	-0.0007	0.0005	0.0010	0.0014	-0.0001	-0.0004
2 0 1	-0.0001	0.0001	0.0001	0.0001	-0.0000	-0.0000
0 0 0	0.0002	0.	0.	0.	-0.0001	0.
1 0 0	-0.0001	0.0000	0.0000	-0.0000	-0.0000	0.0000
-3 -1 0	-0.0000	0.0001	-0.0000	-0.0000	-0.0000	-0.0000
-2 -1 0	-0.0001	0.0002	-0.0001	-0.0000	-0.0000	0.0000
-1 -1 0	-0.0003	0.0010	-0.0010	-0.0003	-0.0000	0.0000
0 -1 0	0.0000	0.0000	0.0172	-0.0005	-0.0027	0.0109
1 -1 0	-0.0017	-0.0442	-0.0442	0.0017	-0.0000	0.0000
2 -1 0	-0.0004	-0.0108	-0.0054	0.0002	-0.0000	0.0000
3 -1 0	-0.0001	-0.0030	-0.0010	0.0000	-0.0000	0.0000
4 -1 0	-0.0000	-0.0009	-0.0002	0.0000	0.0000	-0.0000
5 -1 0	-0.0000	-0.0003	-0.0001	0.0000	-0.0000	0.0000
6 -1 0	-0.0000	-0.0001	-0.0000	0.0000	-0.0000	0.0000
0 -2 0	0.0000	0.0000	0.0001	-0.0000	-0.0000	0.0000
1 -2 0	-0.0000	-0.0002	-0.0002	0.0000	-0.0000	-0.0000

Table D-20

Perturbations of Pluto Due to Earth.

i	j	n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
			cos	sin	cos	sin	cos	sin
0	0	1	-0.0000	0.	0.	0.	0.0002	0.
1	0	1	-0.0001	-0.0002	-0.0005	0.0003	-0.0005	-0.0010
2	0	1	-0.0000	-0.0000	-0.0000	0.0000	-0.0001	-0.0001
0	0	0	0.0004	0.	0.	0.	-0.0003	0.
1	0	0	0.0007	0.0008	0.0000	-0.0000	-0.0000	0.0000
2	0	0	0.0003	0.0003	0.0001	-0.0002	0.0001	-0.0001
3	0	0	0.0001	0.0001	0.0000	-0.0000	0.0000	-0.0000
-3	-1	0	-0.0000	0.0001	-0.0000	-0.0000	-0.0000	-0.0000
-2	-1	0	-0.0001	0.0005	-0.0002	-0.0000	-0.0000	-0.0000
-1	-1	0	-0.0002	0.0022	-0.0022	-0.0002	0.0000	0.0000
0	-1	0	0.0000	0.0000	0.0253	-0.0149	0.0031	0.0225
1	-1	0	-0.0392	-0.0642	-0.0642	0.0392	0.0000	-0.0000
2	-1	0	-0.0096	-0.0157	-0.0078	0.0048	0.0000	0.0000
3	-1	0	-0.0026	-0.0043	-0.0014	0.0009	0.0000	0.0000
4	-1	0	-0.0008	-0.0012	-0.0003	0.0002	-0.0000	-0.0000
5	-1	0	-0.0002	-0.0004	-0.0001	0.0000	0.0000	0.0000
6	-1	0	-0.0001	-0.0001	-0.0000	0.0000	0.0000	0.0000
0	-2	0	-0.0000	0.0000	0.0002	-0.0001	0.0000	0.0002
1	-2	0	-0.0003	-0.0005	-0.0005	0.0003	-0.0000	0.0000
2	-2	0	-0.0001	-0.0001	-0.0001	0.0000	-0.0000	0.0000

Table D-21

Perturbations of Pluto Due to Mars.

i	j	n	$\alpha \cdot 10^6$		$\beta \cdot 10^6$		$\gamma \cdot 10^6$	
			cos	sin	cos	sin	cos	sin
1	0	1	0.0000	-0.0003	-0.0005	-0.0000	-0.0001	-0.0003
0	0	0	0.0001	0.	0.	0.	-0.0004	0.
1	0	0	0.0004	-0.0008	-0.0000	-0.0000	-0.0000	0.0000
2	0	0	0.0001	-0.0003	-0.0001	-0.0001	0.0000	-0.0000
3	0	0	0.0000	-0.0001	-0.0000	-0.0000	0.0000	-0.0000
-2	-1	0	0.0001	-0.0001	0.0000	0.0000	-0.0000	-0.0000
-1	-1	0	0.0002	-0.0002	0.0002	0.0002	-0.0000	-0.0000
0	-1	0	-0.0000	0.0000	-0.0044	-0.0018	0.0022	-0.0027
1	-1	0	-0.0045	0.0113	0.0113	0.0045	0.0000	0.0000
2	-1	0	-0.0011	0.0028	0.0014	0.0006	0.0000	0.0000
3	-1	0	-0.0003	0.0008	0.0003	0.0001	0.0000	0.0000
4	-1	0	-0.0001	0.0002	0.0001	0.0000	-0.0000	0.0000
5	-1	0	-0.0000	0.0001	0.0000	0.0000	0.0000	0.0000
0	-2	0	0.0000	-0.0000	-0.0002	-0.0001	0.0001	-0.0001
1	-2	0	-0.0002	0.0005	0.0005	0.0002	-0.0000	-0.0000
2	-2	0	-0.0001	0.0001	0.0001	0.0000	-0.0000	-0.0000

The effects of general relativity on planetary motions are given by de Sitter (Reference D-4). The corrections are represented with sufficient accuracy by the secular motion of the longitude of perihelion as applied by Clemence (References D-1 and D-3). Let $\tilde{\omega}$ be the longitude of perihelion and $\delta\tilde{\omega}$ the relativistic correction. From page 15 a small change in the argument of perihelion can be reflected in the integration constant z_2 by putting

$$z_2 = - \frac{e}{\sqrt{1-e^2}} \delta\tilde{\omega}_1$$

The resulting terms in the perturbation components are

$$\delta\alpha = - \frac{e}{\sqrt{1-e^2}} \delta\tilde{\omega} \sin \epsilon$$

and

$$\delta\beta = - \frac{e}{\sqrt{1-e^2}} \delta\tilde{\omega} \left(2 \cos \epsilon - \frac{e}{2} \cos 2\epsilon \right).$$

The relativistic motion of the perihelion is given by

$$\delta\tilde{\omega} = \frac{3}{c^2} \cdot \frac{n^3 a^2}{(1-e^2)}$$

where c is the velocity of light which is taken to be 173.29 astronomical units per Julian day based on the solar parallax (Reference D-5, page 169). A small secular term in β will result, but this is removed by adjusting the constant z_3 . The resulting change in the constant term of α is too small to affect the tabular values.

Table D-22

Relativistic Corrections to the Motions of the Five Outer Planets. Entries are the Coefficients of $T \sin(i g)$ in α and $T \cos(i g)$ in β .

Planet	i	$\alpha \cdot 10^6$ sin	$\beta \cdot 10^6$ cos
Jupiter	1	-0.0147	-0.0293
	2	-0.0004	-0.0004
Saturn	1	-0.0036	-0.0072
	2	-0.0001	-0.0001
Uranus	1	-0.0005	-0.0011
Neptune	1	0.	-0.0001
Pluto	1	-0.0005	-0.0010
	2	-0.0001	-0.0001

It is of interest to compare the results with those obtained by Clemence. For this purpose the perturbations δx , δy , δz have been computed from the series for α , β , γ due to the inner planets and the relativistic corrections. The largest differences come in the terms affected by the constants of integration and the small terms not computed by Clemence. I have chosen the integration constants so as to eliminate certain terms in the perturbations as is done in the application of Hansen's method with mean elements.

The coefficients are in units of the ninth decimal of an astronomical unit. They are to be multiplied by $T^n \cos, \sin(i g + j g')$ where T

is the time in centuries, g is the mean anomaly of the disturbed planet and g' is the mean anomaly of the disturbing planet. Where i , j or n is not tabulated, the value is to be taken as zero. These

results constitute a confirmation of Clemence's results since, excepting the constants of integration, the individual differences in the coefficients are much less than $0.^{\circ}01$. Tables D-23 through D-27 contain the terms affected by the constants of integration. There is some innaccuracy in the final digits of the coefficients for Uranus, Neptune and Pluto computed by the author due to the conversion from radians to astronomical units.

Table D-23
Perturbations of Jupiter which are Independent
of the Mean Anomalies of the Disturbing Planets.

Computed by	i	n	$\delta x \cdot 10^9$		$\delta y \cdot 10^9$		$\delta z \cdot 10^9$	
			cos	sin	cos	sin	cos	sin
Carpenter	0	1	756	0	-2598	0	-1100	0
	1	1	32	213	17	-61	-445	278
	2	1	221	956	-872	209	-390	74
	3	1	16	69	-64	15	-28	5
	4	1	1	5	-4	1	-2	0
Clemence	0	1	668	0	-2599	0	-1099	0
	1	1	-15	29	185	-140	-375	256
	2	1	222	940	-857	210	-384	75
	3	1	16	68	-62	15	-27	5
	4	1	1	4	-4	1	-2	0
Carpenter	0	0	-12	0	-18	0	-8	0
	1	0	252	-61	58	229	19	100
	2	0	4	-17	16	4	7	2
	3	0	-1	-2	1	-1	1	0
Clemence	0	0	-52	0	-75	0	-32	0
	1	0	246	-58	56	226	18	98
	2	0	12	-3	3	11	1	5

Table D-24
Perturbations of Saturn which are Independent
of the Mean Anomalies of the Disturbing Planets.

Computed by	i	n	$\delta x \cdot 10^9$		$\delta y \cdot 10^9$		$\delta z \cdot 10^9$	
			cos	sin	cos	sin	cos	sin
Carpenter	0	1	769	0	69	0	-22	0
	1	1	56	5	-70	-20	180	215
	2	1	259	-3	5	239	-3	104
	3	1	21	0	1	19	-1	8
	4	1	2	0	0	1	0	1
Clemence	0	1	787	0	38	0	-36	0
	1	1	1	8	-74	-73	177	196
	2	1	262	-7	9	241	-2	106
	3	1	22	-1	1	20	-1	8
	4	1	2	0	0	2	0	1
Carpenter	0	0	-11	0	-18	0	-9	0
	1	0	-7	-137	127	-8	53	3
	2	0	12	4	-3	10	-2	5
	3	0	1	1	-1	1	-1	0
Clemence	0	0	-40	0	-79	0	-34	0
	1	0	-4	-135	125	-5	52	4
	2	0	0	-8	7	0	3	0

Table D-25

Perturbations of Uranus which are Independent
of the Mean Anomalies of the Disturbing Planets.

Computed by	i	n	$\delta x \cdot 10^9$		$\delta y \cdot 10^9$		$\delta z \cdot 10^9$	
			cos	sin	cos	sin	cos	sin
Carpenter	0	1	43	0	98	0	42	0
	1	1	1	-9	3	-2	4	10
	2	1	-2	-39	35	-2	15	0
	3	1	0	-3	2	0	1	0
Clemence	0	1	22	0	112	0	49	0
	1	1	0	-1	-2	-3	2	8
	2	1	7	-40	37	7	16	3
	3	1	1	-3	3	0	1	0
Carpenter	0	0	-9	0	-16	0	-7	0
	1	0	-66	-9	9	-60	5	-26
	2	0	-9	-21	19	-8	8	-3
	3	0	-1	-2	1	-1	1	0
Clemence	0	0	-37	0	-72	0	-31	0
	1	0	-66	-12	11	-60	6	-26
	2	0	-3	-1	0	-3	0	-1

Table D-26

Perturbations of Neptune which are Independent
of the Mean Anomalies of the Disturbing Planets.

Computed by	i	n	$\delta x \cdot 10^9$		$\delta y \cdot 10^9$		$\delta z \cdot 10^9$	
			cos	sin	cos	sin	cos	sin
Carpenter	0	1	51	0	-31	0	-14	0
	1	1	0	1	3	-4	-8	11
	2	1	14	16	-14	13	-6	5
Clemence	no terms given							
Carpenter	0	0	-9	0	-15	0	-9	0
	1	0	29	-30	29	27	11	12
	2	0	-17	9	-8	-15	-3	-7
Clemence	0	0	-40	0	-72	0	-31	0

Table D-27

Perturbations of Pluto which are Independent
of the Mean Anomalies of the Disturbing Planets.

Computed by	i	n	$\delta x \cdot 10^9$		$\delta y \cdot 10^9$		$\delta z \cdot 10^9$	
			cos	sin	cos	sin	cos	sin
Carpenter	0	1	18	0	37	0	15	0
	1	1	-6	-30	14	19	-24	-58
	2	1	-14	1	-4	-10	-1	-8
	3	1	-4	1	-2	-4	1	-2
	4	1	-2	1	-1	-1	0	-1
Clemence	no terms given							
Carpenter	0	0	-17	0	-2	0	-34	0
	1	0	-19	11	-23	-8	-2	-6
	2	0	-15	13	-13	-15	5	-13
	3	0	-6	3	-3	-6	1	-3
	4	0	-2	1	-1	-2	0	-1
Clemence	0	0	-40	0	-72	0	-31	0

Table D-28

Perturbations of the Outer Planets Due to the Inner Planets:
Terms Depending on the Mean Anomalies of the Inner Planets as Computed
by Clemence. The Argument is jg' where g' is the Mean Anomaly of the
Disturbing Planet. The Values are to be Compared with those Computed
by the Author in the Following Tables.

Disturbing Planet	j	$\delta x \cdot 10^9$		$\delta y \cdot 10^9$		$\delta z \cdot 10^9$	
		cos	sin	cos	sin	cos	sin
Mercury	-1	15	61	55	-11	28	-12
	-2	1	6	5	-1	3	-1
Venus	-1	-1157	1339	1196	1089	612	406
	-2	-4	5	4	4	2	1
Earth	-1	-619	2971	2727	567	1183	246
	-2	-5	24	22	5	10	2
Mars	-1	443	-209	-186	-404	-97	-180
	-2	21	-10	-9	-19	-5	-8
	-3	2	-1	-1	-2	0	-1

Table D-29

Perturbations of Jupiter Computed by Carpenter from Table D-2 Through D-5
The Argument is $(ig + jg')$ where g and g' are the Mean Anomalies
of Jupiter and the Disturbing Planet Respectively.

Disturbing Planet	i	j	$\delta x \cdot 10^9$		$\delta y \cdot 10^9$		$\delta z \cdot 10^9$	
			cos	sin	cos	sin	cos	sin
Mercury	0	-1	15	60	55	-11	28	-12
	0	-2	2	6	6	-1	3	-1
	0	-3	0	1	1	0	0	0
Venus	0	-1	-1157	1339	1195	1089	612	406
	0	-2	-4	4	4	4	2	1
Earth	0	-1	-627	2970	2725	575	1182	249
	2	-1	0	1	-1	0	0	0
	3	-1	1	0	0	1	0	0
	0	-2	-5	25	23	5	10	2
	1	-2	-1	0	0	1	0	0
	3	-2	-2	1	-1	-2	0	-1
	4	-2	-1	0	0	-1	0	0
	0	-3	0	1	0	0	0	0
	0	-1	445	-208	-185	-406	-97	-180
Mars	2	-1	1	-2	2	1	1	0
	3	-1	1	6	-6	1	-3	0
	4	-1	-1	3	-2	0	-1	0
	5	-1	-1	1	-1	0	0	0
	6	-1	0	1	-1	0	0	0
	0	-2	21	-10	-9	-19	-5	-8
	3	-2	0	-3	3	0	1	0
	0	-3	1	-1	-1	-1	0	0
	4	-3	0	-1	0	0	0	0

Table D-30

Perturbations of Saturn from Tables D-6 Through D-9.

Disturbing Planet	i	j	$\delta x \cdot 10^9$		$\delta y \cdot 10^9$		$\delta z \cdot 10^9$	
			cos	sin	cos	sin	cos	sin
Mercury	0	-1	15	61	55	-11	28	-12
	0	-2	2	6	5	-1	3	-1
	0	-3	0	1	1	0	0	0
Venus	0	-1	-1157	1339	1196	1089	612	406
	0	-2	-4	5	4	4	2	1
Earth	0	-1	-627	2970	2724	576	1182	249
	0	-2	-5	25	23	4	10	2
Mars	0	-1	444	-208	-185	-405	-97	-180
	0	-2	20	-10	-9	-19	-4	-8
	0	-3	1	-1	-1	-1	0	0

Table D-31
Perturbations of Uranus from Tables D-10 Through D-13.

Disturbing Planet	i	j	$\delta x \cdot 10^9$		$\delta y \cdot 10^9$		$\delta z \cdot 10^9$	
			cos	sin	cos	sin	cos	sin
Mercury	0	-1	15	62	55	-11	28	-12
	1	-1	0	0	1	0	0	0
	0	-2	1	6	5	-1	2	0
	0	-3	0	2	2	0	1	0
Venus	0	-1	-1157	1341	1195	1090	611	406
	1	-1	0	1	0	0	0	0
	2	-1	-1	-1	-1	1	0	0
	0	-2	-3	4	4	3	2	1
Earth	0	-1	-627	2968	2725	575	1182	250
	1	-1	0	-1	0	0	0	0
	2	-1	1	0	0	-1	0	0
	3	-1	0	-1	0	0	0	0
	0	-2	-5	24	22	5	10	2
	1	-2	1	0	0	0	0	0
	3	-2	0	1	-1	0	0	0
	0	-3	445	-209	-185	-406	-98	-181
Mars	1	-1	1	0	0	0	0	0
	3	-1	0	1	-1	0	0	0
	4	-1	1	0	0	0	0	0
	0	-2	20	-9	-8	-18	-4	-8
	3	-2	1	0	0	1	0	0
	0	-3	2	0	0	-2	0	-1

Table D-32
Perturbations of Neptune from Tables D-14 Through D-17.

Disturbing Planet	i	j	$\delta x \cdot 10^9$		$\delta y \cdot 10^9$		$\delta z \cdot 10^9$	
			cos	sin	cos	sin	cos	sin
Mercury	0	-1	14	62	55	-10	29	-12
	0	-2	2	6	6	-2	2	-1
Venus	-1	-1	0	-1	0	0	0	0
	0	-1	-1159	1339	1196	1089	613	406
	1	-1	0	-1	0	1	0	1
	3	-1	-1	0	0	-1	0	0
Earth	0	-2	-4	4	4	4	2	2
	-1	-1	-1	0	0	0	0	0
	0	-1	-625	2971	2726	575	1181	249
	1	-1	1	0	1	0	0	0
	3	-1	-1	0	0	-1	0	0
Mars	0	-2	-5	25	23	5	10	1
	0	-1	442	-207	-184	-405	-96	-180
	1	-1	0	-1	0	1	0	0
	3	-1	1	1	-1	0	0	0
	0	-2	19	-10	-9	-18	-4	-7
	0	-3	2	-2	-2	-2	-1	-1

Table D-33

Perturbations of Pluto from Tables D-18 Through D-21.

Disturbing Planet	i	j	$\delta x \cdot 10^9$		$\delta y \cdot 10^9$		$\delta z \cdot 10^9$	
			cos	sin	cos	sin	cos	sin
Mercury	-1	-1	-1	1	0	1	0	0
	0	-1	15	61	56	-8	26	-12
	1	-1	0	-1	0	-1	0	0
	2	-1	0	0	0	1	0	0
	3	-1	-1	-1	-2	-1	0	0
	4	-1	1	0	0	1	0	0
	-1	-2	-1	0	0	1	0	0
	0	-2	0	5	5	0	2	-1
	1	-2	0	-2	0	0	0	1
Venus	-4	-1	1	-1	-1	-1	-1	0
	-3	-1	1	2	1	0	0	-1
	-2	-1	-1	0	0	-1	0	0
	-1	-1	0	1	0	0	0	0
	0	-1	-1160	1340	1195	1087	613	407
	1	-1	-1	0	-1	0	0	0
	2	-1	-1	0	0	1	0	0
	3	-1	-1	0	0	1	0	0
	4	-1	0	1	1	2	1	0
	5	-1	0	1	-1	0	0	0
	6	-1	0	0	0	-1	0	0
	7	-1	0	1	0	1	0	0
	0	-2	-5	5	5	6	3	0
	1	-2	1	-1	-1	-1	-1	0
Earth	-4	-1	0	-1	0	-1	0	0
	-3	-1	2	1	0	-1	-1	-1
	-2	-1	0	1	-2	1	-1	0
	-1	-1	0	1	-1	1	0	0
	0	-1	-627	2969	2726	576	1185	252
	1	-1	1	-1	0	-1	0	0
	2	-1	1	-1	-1	-1	0	0
	3	-1	2	0	0	0	-1	0
	4	-1	-2	1	-2	2	0	0
	5	-1	1	-1	1	-1	0	0
	6	-1	0	-1	-1	0	0	0
	7	-1	1	-1	1	1	0	0
	0	-2	-5	24	21	5	8	3
	2	-2	0	-1	-1	0	0	0
	3	-2	0	-2	2	0	0	1
Mars	-3	-1	-2	0	0	2	1	1
	-2	-1	1	-1	1	-1	0	0
	-1	-1	0	1	-1	0	0	0
	0	-1	444	-211	-183	-405	-97	-180
	1	-1	2	-1	0	-2	0	0
	2	-1	1	-1	-1	-1	-1	0
	3	-1	-1	0	-1	-1	0	0
	4	-1	0	-1	-1	0	0	0
	5	-1	1	1	0	2	0	0
	6	-1	-1	0	0	-1	0	0
	0	-2	20	-10	-8	-18	-4	-7
	1	-2	1	0	1	0	0	0
	2	-2	-1	0	0	1	0	0
	3	-2	2	0	1	2	0	1

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